Summary of the PhD thesis

Uniform convergence of trigonometric series with p-bounded variation coefficients

written by

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This dissertation is devoted to the study of the uniform convergence of trigonometric series. Our main aim is to extend and generalize the known results in literature concerning this issue. The starting point are results characterizing the uniform convergence of single and double trigonometric series, where the important assumption is the monotonicity of coefficients of these series. The first result referring to this issue was published in 1916 [1], where T. W. Chaundy and A. E. Jolliffe characterized the uniform convergence of sine series using the monotonicity of the coefficients of this series. A similar result for single cosine series was presented in 1959 in the monograph written by A. Zygmund [20]. Next, in 1966 I. E. Žak and A. A. Šneider [18] obtained this result for the uniform convergence of double sine series, where the coefficients form double non-increasing sequences.

A significant moment in the research on the uniform convergence of sine series is the paper of L. Leindler [13] in 2001, where a new class of single sequences of rest bounded variation is introduced. It has become an inspiration to generalize and extend classical results regarding to uniform convergence of trigonometric series by replacing monotonicity sequences with sequences of bounded variation. New classes have been considered

in the papers: [7], [15], [16], [17], [19]. It is worth to recall B. Szal paper [15], where the class of single sequences of bounded variation was extended by replacing the sum of differences of terms of the sequence with step one by the sum of difference of terms of the sequence by the step $r \in \mathbb{N}$. Moreover, S. Tikhonov in paper [17] and E. Liflyand in paper [14] defined a new class of bounded variation sequences, where the differences of the terms of sequences are raised to p power and then the p index of the radical from the whole sum is calculated. In 2009 in paper [4] P. Kórus and F. Móricz introduced the class of double sequences of bounded variation for which the generalized result was obtained by I. E. Žak and A. A. Šneider. Next, P. Kórus in paper [5] generalized this result for another, wider class of double sequences of bounded variation. In the same paper, he presented the theorem about the uniform convergence of double mixed series, where coefficients of this series form the sequences belonging to those classes. Moreover, the classes of double sequences of bounded variation were studied in papers: [2], [3], [6].

In the PhD dissertation we have introduced a new, wider class of single sequences of p-bounded variation, which is the generalization of the class considered by S. Tikhonov [17], L. Leindler [13] and B. Szal [15]. Analogically, new classes of double sequences of p-bounded variation are introduced here. These classes of such type are used in the dissertation to generalize and extend the known results related to the uniform convergence of single, double and mixed trigonometric series.

The thesis consists of three chapters. In Chapter 1 we present selected classes of single and double sequences of bounded variation. At the beginning of Subchapter 1.1 we define a new class of p-bounded variation sequences. Next, we present some basic classes of single sequences of bounded variation and we study the relations between those classes. In the further part of this subchapter we show that the new class is wider than the classes presented in the aforementioned literature. Subchapter 1.2 is devoted to double sequences of bounded variation and has an analogous structure to Subchapter 1.1. We introduce new class of double sequences of p-bounded variation and we show that this class is wider than the classes considered in the literature. This Chapter is based on articles: [10], [8].

Chapter 2 is devoted to the uniform convergence of single sine and cosine series. The Subchapter 2.1 has an introductory character and supplies some auxiliary theorems used in this Chapter. In Subchapter 2.2 we present the main results concerning to the uniform convergence of sine and cosine series, where we assume that the coefficient of these series belongs to a new class of sequences defined in Subchapter 1.1. These theorems generalize and

extend the results obtained by K. Duzinkiewicz in paper [3] and consequently the earlier results. This Chapter is based on articles: [10], [11].

Chapter 3 contains the theorems about uniform convergence of double sine series and double mixed series. Analogous to Chapter 2, Subchapter 3.1 supplies some auxiliary theorems used in a further part of this Chapter. In the first part of Subchapter 3.2 we prove theorems concerning to the uniform convergence of double sine series, which coefficients form the sequence belonging to the class introduced in Subchapter 1.2. Moreover, we present some remarks showing that these theorems generalize and extend results achieved by K. Duzinkiewicz and B. Szal in paper [2]. In the further part of this subsection we present results on the uniform convergence of double mixed series, where we assume that the coefficients of these series belongs to the class of sequences defined in Subchapter 1.2. At the end, we show that these theorems generalize and extend the results obtained by K. Duzinkiewicz in paper [3] and consequently the earlier result achieved by P. Kórus and F. Móricz in paper [4]. This Chapter is based on articles: [9], [8].

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