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Analysis of Boundedness and Safeness in a Petri Net-Based Specification of Concurrent Control Systems

Analiza ograniczoności i bezpieczeństwa współbieżnych systemów sterowania specyfikowanych sieciami Petriego

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I dedicate the dissertation to my beloved fiancée and
to bl. Carlo Acutis, who is the patron of this work.

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Abstract

Control systems can be found in many areas of everyday life, such as banking, medical care, manufacturing, transportation, and entertainment. Their intensive development requires designers to use advanced and multi-functional tools to support the process of their design. Petri nets are one of such modeling approaches. They are increasingly popular, available for analysis, and easily yield to graphic design. Therefore, analyzing boundedness and safeness of systems specified by Petri nets becomes an important challenge. This dissertation provides an extensive overview of applications and algorithms that analyze boundedness and safeness of Petri nets. Boundedness signifies a finite number of reachable states of a control system, whereas safeness characterizes a binary behavior which, combined with a set of logical input and output signals, is used in Petri nets that can be easily implemented in configurable FPGAs. In the absence of sufficiently efficient and effective analytical methods of Petri net properties, due to their exponential computational complexity, novel algorithms are proposed to fill the gap in this area. The introduced solutions are described in detail and supported by a series of experimental findings form a set of 243 Petri nets. Their limitations are also discussed. In addition, a real-life manufacturing control system described on a Petri net, which illustrates the most important benefits of designing concurrent systems by means of bounded and safe nets, was prepared.

Keywords: Petri net-based specification, concurrent control system, boundedness of Petri nets, safeness of Petri nets, verification of Petri net properties.

Streszczenie

Systemy sterujące odnajdujemy w wielu dziedzinach codziennego życia, takich jak np. bankowość, ochrona zdrowia, produkcja, transport czy rozrywka. Intensywny ich rozwój wymaga od projektantów zastosowania zaawansowanych i wielofunkcyjnych narzędzi wspomagających projektowanie. Jednym z możliwych sposobów modelowania są sieci Petriego. Cechują się one coraz większą popularnością, dostępnością metod analizy, a także możliwością graficznego projektowania. Wynika stąd ważne wyzwanie jakim okazuje się analiza ograniczoności i bezpieczeństwa systemów specyfikowanych sieciami Petriego. Niniejsza praca dokonuje obszernego przeglądu zastosowań oraz algorytmów analizujących ograniczoność i bezpieczeństwo sieci Petriego. Ograniczoność odpowiada za skończoną liczbę osiągalnych stanów systemu sterowania. Natomiast bezpieczeństwo nadaje sieci binarne zachowanie, które w połączeniu ze zbiorem sygnałów logicznych wejściowych i wyjściowych znajduje zastosowanie w sieciach Petriego, które z łatwością mogą być zaimplementowane w konfigurowalnych układach FPGA. Wobec braku wystarczająco efektywnych i skutecznych metod analizy właściwości sieci ze względu na wykładniczą złożoność obliczeniową w przypadku ogólnym, zaproponowane zostają nowe algorytmy uzupełniające niedostateczności w tym zakresie. Wprowadzone rozwiązania są szczegółowo opisane oraz poparte szeregiem badań eksperymentalnych przeprowadzonych na zbiorze 243 sieci Petriego. Omówiono także ich ograniczenia. Ponadto przygotowano rzeczywisty przemysłowy system sterujący opisany na sieci Petriego, który obrazuje najważniejsze korzyści projektowania systemów współbieżnych z użyciem ograniczonych i bezpiecznych sieci.

Słowa kluczowe: specyfikacja siecią Petriego, współbieżny system sterowania, ograniczoność sieci Petriego, bezpieczeństwo sieci Petriego, weryfikacja właściwości sieci Petriego.

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Glossary

Assumed time	it is maximal allowed reasonable run-time of algorithms; in this dissertation, the assumed time is fixed to one hour ($3.6 \cdot 10^6$ milliseconds); this term is closely related to efficiency .
Effectiveness	in relation to algorithms, it means that the obtained results are correct. In other words, the method always produces correct answers after a finite number of steps .
Efficiency	in relation to algorithms, it means that the results can be found within the assumed time .
Toolchain	a set of algorithms that are executed in a specific order for particular cases in order to achieve efficient and effective analysis .

Acronyms

ACN	Asymmetric Choice Net.
BDD	Binary Decision Diagram.
BRG	Basis Reachability Graph.
CAD	Computer-Aided Design.
CPS	Cyber-Physical System.
EFC	Extended Free-Choice net.
FCN	Free-Choice Net.
FMS	Flexible Manufacturing System.
FPGA	Field-Programmable Gate Array.
FSM	Finite State Machine.
GPU	Graphics Processing Unit.
ILPP	Integer Linear Programming Problem.
IOPT	Input-Output Place-Transition net.
IoT	Internet of Things.
LTL	Linear-time Temporal Logic.
MG	Marked Graph.
MIP	Mixed Integer Programming.
NP-hard	Non-deterministic Polynomial-time hardness.
PLC	Programmable Logic Controller.
PN	Petri Net.
PNH	Petri Net Hippo format.

PNML	Petri Net Markup Language.
RAM	Random Access Memory.
SM	State Machine.
SMC	State Machine Component.
UCF	User Constraint File.
UML	Unified Modeling Language.
VHDL	Very high-speed integrated circuit Hardware Description Language.
XML	Extensible Markup Language.

Chapter 1

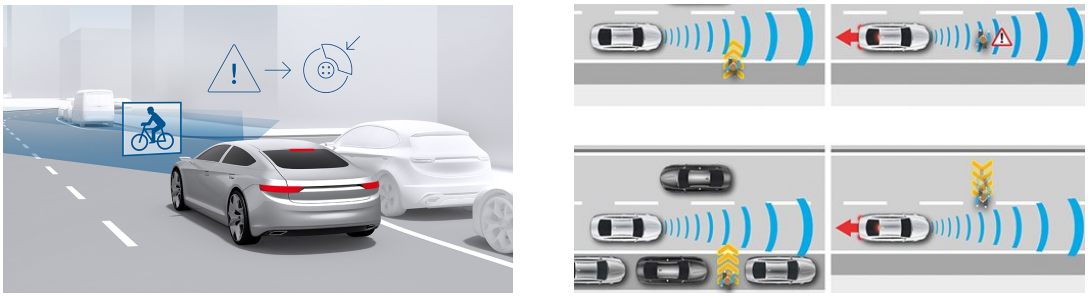
Introduction

Control systems surround people everywhere [155]. Recent years have witnessed wide development of such systems [153]. This is due to growing demand for systems that autonomously control our environment. For instance, there are lamps automatically turning on at dusk, heating systems turning off when we go to work and similar ideas which make our lives easier, cheaper, and more ecological. These so called *smart* solutions can be found in various aspects of human life: including banking [13, 41, 88, 129], manufacturing [32, 78, 94], automotive [135], transportation [135], medical care [11, 31, 35], process mining [2, 15, 28, 62, 68, 105] and others [116].

Control systems can be seen as systems that manage other systems, which are known as operational systems [155]. Such systems send appropriate signals to operational systems based on input values and logic conditions [155]. Whereas a several years ago, there were simple control systems with few input and output signals, in recent years designers have elaborated complex, concurrent, and advanced systems commonly known as: embedded systems [64], Internet of Things (IoT), flexible manufacturing systems (FMS) [4, 9, 10, 100] and most popular nowadays cyber-physical systems (CPS) [7, 8, 71, 77, 91, 93, 152, 157, 159, 164]. Since it is impossible to design such sophisticated systems without the application of computer-aided design (CAD) tools, CAD software [157] is used, e.g., to perform modeling based on finite state machines, UML diagrams, Petri nets, interpreted nets, etc.

Let us imagine a real-life situation and assume that there is a moving car near a playground where children are playing. One child suddenly runs across the road in front of the car. Now consider a modern car with advanced control systems on board. In such a case, the car's front sensors simultaneously detect an object moving into the road. A dedicated control system simultaneously analyzes the input data from the front sensors and decides to send an emergency signal to the operational system. As a result, the car is

automatically halted. Such solutions (illustrated in Fig. 1.1) are offered e.g., by Bosch Mobility Solutions[1] (Fig. 1.1a) or Continental-Automotive[2] (Fig. 1.1b). This way, various control systems make our life easier, cheaper, and also safer.



(a) Automatic emergency braking

(b) Emergency brake assist - pedestrian

Figure 1.1: Examples of safety control systems in modern cars

1.1 Motivation

The real-life automotive example attests to dynamic development and increasing levels of complexity in modern concurrent control systems. It encourages the use of advanced modeling (specifying) approaches [92], one of which is Petri nets, which, in addition to graphic (graph-based) representation, allow for formal description (specification) [155]. Moreover, Petri nets are also supported by formal mechanisms that make it possible to analyze models. Such analyses permit examination of systems' reliability and robustness already at the specification stage [71], which may impact the time and the costs of a designed concurrent control system.

In their very recent research, various scientists have proved the profitability and the usefulness of the idea of Petri nets, e.g., Gomes et al. in [63], Grobelna et al. in [71], Lee and Seshia in [92], Nuño-Sánchez et al. in [113], Ramirez-Trevino et al. in [123], Wiśniewski et al. [155]. Nevertheless, their research also gives rise to an analytical problem regarding Petri nets' properties analysis of Petri nets used for concurrent control system specifications [71, 154, 155, 157, 160, 168, 169]. Such analyses of Petri net properties can be performed based on state space exploration or by means of a linear algebraic approach (solving linear equations) [166, 168, 169]. The former approach examines a state explosion problem [122, 167], which means that as systems grow larger, the number of all possible states grows exponentially. In the linear algebraic approach, the number of computed invariants can also grow exponentially [155, 167, 169]. Hence, both approaches are limited by exponential complexity in relation to the time they need

[1] More information available online: https://www.bosch-mobility-solutions.com/en/solutions/assistance-systems/automatic-emergency-braking (accessed on December 11, 2022)

[2] More information available online: https://www.continental-automotive.com/en-gl/Passenger-Cars/Autonomous-Mobility/Functions/Cruising-Driving/Emergency-Brake-Assist/Pedestrian (accessed on December 11, 2022)

to produce results in the general case [101]. This way, results cannot always be obtained in the assumed time. In such a case, we conclude that the selected method is not efficient [167].

Boundedness and safeness are the most important properties of Petri net-based specifications [168, 169]. Boundedness is responsible for the finite number of reachable states of a control system [85, 168]. In other words, designers expect systems whose behavior can be controlled and all their states can be determined. Therefore, there should be no states of the system that cannot be predicted in the specification. Hence, the Petri nets used in formal specifications of concurrent control systems are required to be bounded [167–169]. In addition, there is sometimes a need to use safe (*stronger* boundedness) Petri nets [85, 169]. Safe Petri nets are capable of logical control, i.e., they are able to indicate whether an action should be active or inactive (on/ off nature) [155].

1.2 Thesis and Main Purposes

The following thesis has been formulated for this dissertation: *Analysis of boundedness and safeness of concurrent control systems specified by Petri nets can be performed effectively and efficiently.* Furthermore, the dissertation has the following purposes:

- to conduct a broad survey on the boundedness and safeness of Petri net properties;
- to propose novel algorithms for analyzing boundedness;
- to propose novel algorithms for analyzing safeness;
- to verify effectiveness and efficiency of the proposed methods.

According to classical definition, efficiency of an algorithm means that results can be found within the assumed time [167]. In this work, the assumed time is fixed to one hour. Furthermore, effectiveness of an algorithm means that the obtained results are correct [167]. In other words, the results are consistent with the definition of the property under test.

1.3 Structure

The first chapter of this dissertation includes introduction to the subject matter, formulation of its thesis as well as motivation behind its creation. Basic concepts of the theory of Petri nets are introduced in the second chapter, which features a discussion of their most important properties, such as boundedness and safeness. An overview of the literature as well as of the available analysis methods are presented in the third chapter, together with formulation of problems related to checking properties. The fourth chapter proposes novel ideas for procedural analysis and presents preliminaries which are indispensable in

understanding the ideas. Contrary to widely recognized algorithms under some assumptions, the proposed solutions are efficient. Moreover, the novel methods are applied to a real-life large-scale system, while the knowable algorithms are more theoretical ideas. Subsequently, experimental results focusing on effectiveness and efficiency are presented in the fifth chapter. When the Petri nets theory with the proposed innovative analysis solutions are explicated, a case-study of manufacturing control specification based on Petri net is introduced in the sixth chapter. Finally, the seventh chapter summarizes the dissertation and confirms its thesis.

Chapter 2

Theoretical Aspects of Petri Nets

Petri nets is one of the forms of graphical specification and representation of various concurrent control systems [17, 30, 40, 65, 97, 113, 124, 139, 156]. Wide application of Petri nets can be found in the field of distributed systems [43, 85, 171], embedded systems [64], edge computing [154], manufacturing systems [4, 9, 10, 100], as well as in cyber-physical systems [7, 8, 77, 91, 93, 152, 157, 159–162, 164]. They are a useful modeling approach because they are supported by verification, validation, and analytical methods [29, 59, 69, 74, 85, 91, 123, 138, 168, 171]. This way, system designers are able to verify the robustness and reliability of their projected systems [3, 5, 6, 102, 108, 176]. This chapter opens with a discussion on computational complexity of algorithms, as algorithms can be seen as procedures solving computational problems. Subsequently, there are examples of graphs as an introduction to Petri nets. Finally, underlying definitions and notations related to Petri nets and their fundamental properties are presented. Chapter 3 discusses theories and definitions which illustrate in detail contemporarily available algorithms (supported by simple examples). Chapter 4 addresses novel methods for analyzing the properties of boundedness and safeness.

2.1 Algorithms

First of all, the analysis of such properties of Petri nets as boundedness and safeness can be treated as a computational problem to be solved. *An algorithm* can be viewed as a method for solving a well-specified computational problem [80]. Formally, an algorithm is any well-defined computational procedure that takes a set of values as *input* and gives back a set of values as *output* [80]. Therefore, it is a sequence of steps that convert input into output. In our considerations, Petri nets constitute the input to algorithms and the output will result from the analysis of a relevant net property.

Example 2.1

Let us consider a simple computational problem of absolute value computation as an example. Algorithm 2.1 proposes a method to solve the problem. We have the following input set of real numbers: $S_{in} = \{x_1, x_2, x_3, \cdots, x_n\}$ and expect the following output set: $S_{out} = \{|x_1|, |x_2|, |x_3|, \cdots, |x_n|\}$. Firstly, an empty set is assigned to S_{out}. Then, for each number x of the input set, the value is calculated: $\sqrt{x^2}$ and added to the output set S_{out}. Finally, the output set contains absolute values of the numbers from the input set.

Data: set $S_{in} = \{x_1, x_2, x_3, \cdots, x_n\}$
Result: set $S_{out} = \{|x_1|, |x_2|, |x_3|, \cdots, |x_n|\}$
1 $S_{out} \leftarrow \emptyset$;
2 **foreach** *variable x of set* S_{in} **do**
3 | add $\sqrt{x^2}$ to S_{out} ;
4 **end**

Algorithm 2.1: An algorithm example — calculation of the absolute value

2.2 Computational Complexity

Each computer algorithm can be characterized by its computational complexity. Computational complexity is necessary to estimate the efficiency of a method. Below are some preliminaries related to computational complexity of algorithms [164], which we will describe in subsequent chapters of this dissertation.

Definition 2.1. *Time complexity* of an algorithm is the function $f : \mathbb{N} \to \mathbb{N}$ such that $f(n)$ is the maximum number of iteration that the algorithm uses on any input of length n.

Definition 2.2. *An upper bound* for $f(n)$ is $g(n)$ that $f(n) = O(g(n))$.

Definition 2.3. An algorithm is bounded by *a polynomial* in the size of inputs n if the number of its iterations is estimated as

$$f(n) = O(n^c), \tag{2.1}$$

where $c > 0$.

Definition 2.4. An algorithm is bounded by *an exponential* in the size of inputs n if the number of its iterations is estimated as

$$f(n) = O(c^n), \tag{2.2}$$

where $c > 1$.

Definition 2.5. *A polynomial complexity (polynomial time)* of an algorithm indicates that the total run-time of the algorithm to generate all the outputs is bounded by a polynomial in the size of the input.

Definition 2.6. *An exponential complexity (exponential time)* of an algorithm means that the total run-time of the algorithm to generate all outputs is bounded by an exponential in the size of inputs.

Example 2.2

The computational complexity of Algorithm 2.1 can be estimated as follows. The *foreach* loop on lines 2–3 is performed $n = |S_{in}|$ times. Hence, the algorithm is bounded by the function $O(n)$, where n is the number of elements in the input set S_{in}. It is said that this algorithm is characterized by polynomial (first degree) computational complexity.

> Efficiency and Effectiveness

Computer science algorithms are usually measured in two directions: the time of the execution or the memory usage by the program. The *efficiency* term in this dissertation refers to optimization (minimization) of the run-time execution concerning the response in the assumed time. Thus, it does not include other aspects. The focus on improvements is just to reduce running time while maintaining correct results (*effectiveness*).

2.3 Graphs

Before we define Petri nets, we shall introduce some basic concepts and notations related to graph theory [81, 84, 107, 131].

Definition 2.7. *A graph* is defined by a pair

$$G = (V, \mathbb{E}), \tag{2.3}$$

where

- $V = \{v_1, v_2, \cdots, v_n\}$, is a finite, nonempty set of vertices,
- $\mathbb{E} = \{E_1, E_2, \cdots, E_m\}$, is a finite set of *unordered* pair of vertices, called edges.

Definition 2.8. *A digraph (directed graph)* is a pair

$$D = (V, \mathbb{E}), \tag{2.4}$$

where

- $V = \{v_1, v_2, \cdots, v_n\}$, is a finite, nonempty set of vertices,
- $\mathbb{E} = \{E_1, E_2, \cdots, E_m\}$, is a finite set of an *ordered* pair of vertices, called edges.

Example 2.3

The usual way to illustrate a graph is by drawing a circle corresponding to each vertex, joining by a line for an edge and an arrow that informs of the direction (only in case of directed graphs). Figure 2.1 shows a simple digraph that consists of six vertices: $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ and seven edges: $\mathbb{E} = \{E_1, E_2, E_3, E_4, E_5, E_6, E_7\}$. Each edge forms a set of ordered pairs of vertices: $E_1 = \{v_1, v_2\}$, $E_2 = \{v_2, v_3\}$, $E_3 = \{v_3, v_4\}$, $E_4 = \{v_4, v_1\}$, $E_5 = \{v_4, v_5\}$, $E_6 = \{v_6, v_4\}$, $E_7 = \{v_5, v_6\}$.

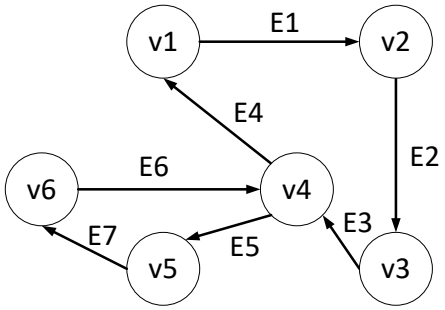


Figure 2.1: An example of a directed graph

2.4 Petri Net-Based Specification

Algorithms in the computer science area can be represented in several ways [160]. The popular ones base on block diagrams or concurrency flowcharts supported by mathematical equations [160]. Nowadays, it looks that more capable methods apply dedicated modeling languages, for instance UML [18, 158] or Petri nets [66, 103]. In this dissertation, we rely on Petri nets because they result in several benefits over named here other approaches [154, 160, 165]. Firstly, the Petri net specifying technique perfectly exploits the parallel operations of a specification [64, 116, 154, 160, 165]. It is important, since most of the operations in concurrent control systems as cyber-physical systems are performed simultaneously [154, 160, 164, 165]. Furthermore, various verification, validation, and analysis tools supplement Petri nets [1, 97, 100, 123, 124, 126, 148, 170, 171]. Methods are used in order to ensure that the formal specification is correct [69, 70]. The correctness of the model is provided before going on with the next steps of the design path [136, 160]. This advantage is especially significant in industry where each return to previous stages is very costly [136]. Sometimes prototyping of concurrent control systems involves splitting of the system into separate subsystems [154, 155, 160, 165]. Petri nets can be automatically decomposed into a set of modules [155]. Additionally, Pereira et al. [118]

proposed a comfortable prototyping framework that includes a graphical editor for Petri net modeling, a state space generator (verification) and a manual simulator (validation).

The idea of Petri nets was introduced by Carl Adam Petri in his dissertation in 1962 [25, 58, 84, 121]. A net called later Petri net [112] is a directed graph where vertexes are places and edges function as transitions. As a digraph, a Petri net can be converted into another strict mathematical notation [25], such as incidence matrix [16]. Thus, we can use the graph theory and linear algebra for various analyses of Petri nets (models of the system). It is impossible in other approaches, such as UML design [18], where we do not have a mathematical structure. Petri nets are still being developed and applied to various disciplines of science and industrial uses [44, 47, 56, 76, 96, 111, 112, 115, 116, 140, 151]. This makes it necessary to modify and introduce their new types, e.g., hybrid Petri nets [42], timed continuous Petri nets [45], colored Petri nets, ordinary Petri nets, interpreted Petri nets and unbounded Petri nets. The multitude of types attests to the fact that each Petri net contains unique features and thus has its own application in different areas [160].

The subsequent sections will introduce mathematical notations of Petri nets used for concurrent control system modeling. They will make use of the base theory with examples, present the most important structural and behavioral properties of Petri nets, as well as expound on their classes.

2.5 Basic Notations and Definitions

This section introduces a formal description of Petri nets and their definitions. Notations explain common Petri net properties, especially boundedness and safeness. The definitions correspond to the notations presented in [85, 112, 154, 155, 160, 165, 168].

For better understanding, the collections, sets and matrices (if applicable) are denoted in uppercase letters, while single variables in lowercase letters and vectors are marked by right-pointing arrows. The multiplication sign is $\cdot$ and $\times$ is a vector dot product. Moreover, matrix transposition is represented by $^{\mathsf{T}}$. A set of natural numbers, including zero, is denoted by $\mathbb{N}$.

Definition 2.9. *A Petri net* N is a 4-tuple:

$$N = (P, T, F, \overrightarrow{M_0}), \tag{2.5}$$

where

- $P = \{p_1, p_2, \cdots, p_n\}$, is a finite set of places,
- $T = \{t_1, t_2, \cdots, t_m\}$, is a finite set of transitions,
- $F \subseteq (P \times T) \cup (T \times P)$, is a finite set of arcs,
- $\overrightarrow{M_0}$ is an initial marking (state) vector.

Graphically, a Petri net is represented as a directed graph whose vertices correspond to places (circles) and edges to transitions (bold line) [85]. Arcs travel from transitions to their output places and from places to their output transitions. Initially, the marked places are pointed by a token (black spot).

Example 2.4

Below is an illustration a Petri net-based specification with a simple example[1]. Figure 2.2 shows a real-life system responsible for controlling a multi-robot. The example presented in [168] involves pick-and-place movements in order to transport or obtain parts by two arms of the robot.

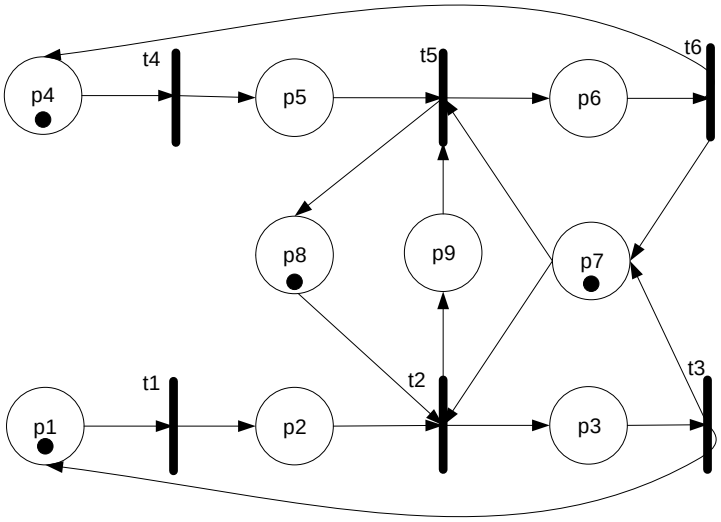


Figure 2.2: Multi-robot Petri net-based specification N_1

Petri net N_1 consists of $|P_1| = 9$ places and $|T_1| = 6$ transitions. There are nine places denoted by $p_1,\ldots,p_9$, and six transitions marked as $t_1,\ldots,t_6$. One token is put on each p_1, p_4, p_7, p_8 place in Fig. 2.2, which means that these places are initially marked: $\overrightarrow{M_0} = [1\ 0\ 0\ 1\ 0\ 0\ 1\ 1\ 0]$.

Definition 2.10. Place p is an *input place* of transition $t \in T$, if $(p,t) \in F$.

Definition 2.11. A set of input places of a transition is defined as:

$$\bullet t = \{p \in P : (p,t) \in F\}. \tag{2.6}$$

Definition 2.12. Place p is an *output place* of transition $t \in T$, if $(t,p) \in F$.

Definition 2.13. A set of output places of a transition is denoted as:

$$t\bullet = \{p \in P : (t,p) \in F\}. \tag{2.7}$$

[1]Available online: http://www.hippo.issi.uz.zgora.pl/index.php?id=petri_net&nr=375 (accessed on December 11, 2022)

Definition 2.14. Transition t is an *input transition* of place $p \in P$, if $(t,p) \in F$.

Definition 2.15. A set of input transitions of a place is defined as:

$$\bullet p = \{t \in T : (t,p) \in F\}. \tag{2.8}$$

Definition 2.16. Transition t is an *output transition* of place $p \in P$, if $(p,t) \in F$.

Definition 2.17. A set of output transitions of a place is denoted as:

$$p\bullet = \{t \in T : (p,t) \in F\}. \tag{2.9}$$

Definition 2.18. A transition t can be *fired* if each of its input places contains a token. Transition firing removes a token from every input place of t and adds a token to every output place of t.

Definition 2.19. A marking (state) $\overrightarrow{M_s}$ is *reachable* from marking $\overrightarrow{M_r}$, if $\overrightarrow{M_s}$ can be obtained from $\overrightarrow{M_r}$ by a finite sequence σ of transition firings.

Definition 2.20. A transition t is *enabled* in a marking $\vec{M}$ if $\forall p \in \bullet t : p \in \vec{M}$.

Example 2.5

Let us look at the multi-robot example once again. Initially, t_1 and t_4 transitions are enabled. After firing transition t_1, the token is moved from its input place p_1 to its output place p_2. Petri net N_1 reaches marking $\overrightarrow{M_1} = [0\ 1\ 0\ 1\ 0\ 0\ 1\ 1\ 0]$. Note that instead of transition t_1 firing, transition t_4 can also be fired from the initial state $\overrightarrow{M_0}$ reaching the state $\overrightarrow{M_2} = [1\ 0\ 0\ 0\ 1\ 0\ 1\ 1\ 0]$. The marking $\overrightarrow{M_1}$ enables transition t_2, whose execution leads to the next markings.

Definition 2.21. Let R be a *reachability graph* that

$$R = (\mathbb{M}, \Sigma), \tag{2.10}$$

where $\mathbb{M}$ is a collection of reachable markings (states) $\vec{M}$ from $\overrightarrow{M_0}$ and Σ a set of fired transitions, representing a full state space of a Petri net.

Example 2.6

Figure 2.3 shows a reachability graph R_1 of a Petri net N_1. In our example of the concurrent control system describing the behavior of a multi-robot, twelve different states can be distinguished in the state space $\mathbb{M}_1$:

$$
\begin{array}{rccccccccc}
 & p_1 & p_2 & p_3 & p_4 & p_5 & p_6 & p_7 & p_8 & p_9 \\
\overrightarrow{M_0} = & [1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0] \\
\overrightarrow{M_1} = & [0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0] \\
\overrightarrow{M_2} = & [1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0] \\
\overrightarrow{M_3} = & [0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0] \\
\overrightarrow{M_4} = & [0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1] \\
\overrightarrow{M_5} = & [0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1] \\
\overrightarrow{M_6} = & [1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1] \\
\overrightarrow{M_7} = & [1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1] \\
\overrightarrow{M_8} = & [0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1] \\
\overrightarrow{M_9} = & [1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0] \\
\overrightarrow{M_{10}} = & [0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1] \\
\overrightarrow{M_{11}} = & [0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0]
\end{array}
. \tag{2.11}
$$

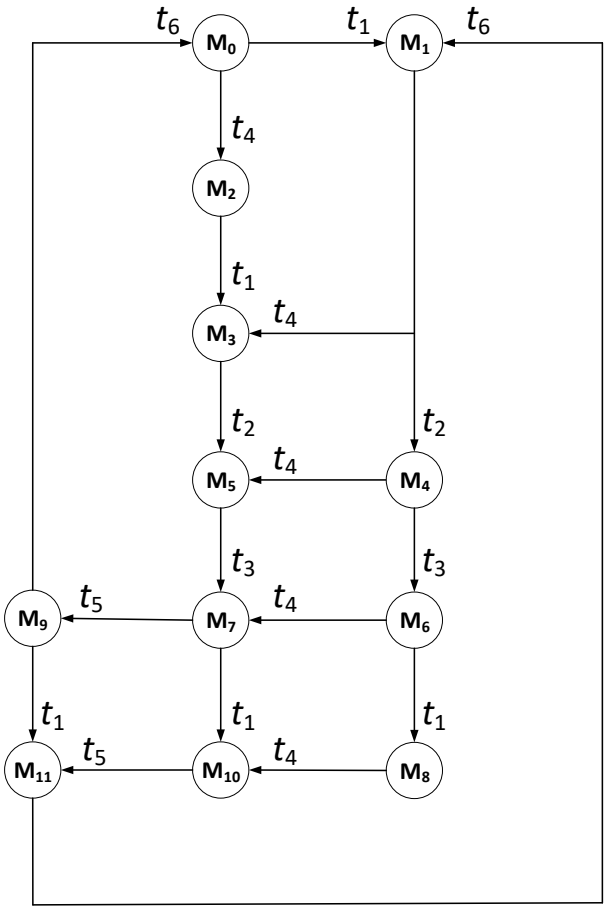


Figure 2.3: The reachability graph of the Petri net N_1

Definition 2.22. A Petri net $N = (P, T, F, \overrightarrow{M_0})$ is claimed to be *k-bounded* or simply *bounded* if

$$\forall p \in P : M(p) \leq k \,, \tag{2.12}$$

where $k \in \mathbb{N}$.

It signifies that there is no reachable marking $\vec{M}$ such that any place $p \in P$ contains more than a finite number k of tokens.

Definition 2.23. A Petri net $N = (P, T, F, \overrightarrow{M_0})$ is said to be *safe* if the number of tokens in each place $p \in P$ does not exceed 1 for any reachable marking $\vec{M}$ that is

$$\forall p \in P : M(p) \leq k = 1 \,. \tag{2.13}$$

A safe Petri net is bounded and 1-bounded by the definition.

Definition 2.24. A Petri net $N = (P, T, F, \overrightarrow{M_0})$ is *L4-live* or just *live* if every transition $t \in T$ can be fired at least once in some firing sequence σ for every reachable marking $\vec{M} \in \mathbb{M}$.

Definition 2.25. A Petri net $N = (P, T, F, \overrightarrow{M_0})$ is *reversible* if for each marking (state) $\vec{M} \in \mathbb{M}$, the initial marking $\overrightarrow{M_0}$ is reachable from that $\vec{M}$.

Definition 2.26. A Petri net $N = (P, T, F, \overrightarrow{M_0})$ is known as *well-formed* if it is *safe (1-bounded)*, live and reversible.

Example 2.7

A state space $\mathbb{M}_1$ of a Petri net N_1 shown in Fig. 2.3 and described in Equation 2.11 points that for each place $p \in P_1$ in every marking $\vec{M} \in \mathbb{M}_1$: $M(p) \leq k = 1$. Therefore, the Petri net N_1 is bounded and safe. Moreover, any transition $t \in T_1$ in any state can be fired in a sequence of firings, thus it is live. In addition, from any reachable marking there is a return path to the initial state $\overrightarrow{M_0}$ through a sequence of firings, which makes the Petri net reversible. The Petri net N_1 is safe, live, and reversible, which proves that it is well-formed.

Definition 2.27. *An interpreted net* is a *well-formed* Petri net $N = (P, T, F, \overrightarrow{M_0})$ with additional sets of input and output signals, defined as 6-tuple:

$$\mathcal{N} = (P, T, F, \overrightarrow{M_0}, X, Y) \,, \tag{2.14}$$

where

- $P = \{p_1, p_2, \cdots, p_n\}$ is a finite set of places,
- $T = \{t_1, t_2, \cdots, t_m\}$ is a finite set of transitions,

- $F \subseteq (P \times T) \cup (T \times P)$ is a finite set of arcs,
- $\overrightarrow{M_0}$ is an initial marking (state) vector,
- $X = \{x_1, x_2, \cdots, x_i\}$ is a finite set of logic inputs,
- $Y = \{y_1, y_2, \cdots, y_j\}$ is a finite set of logic outputs.

In contrast to Petri nets, interpreted nets are supported by additional input and output signals which are used for communication with the environment (real word). Input signals (e.g., received from sensors) are usually related to transitions, while outputs (e.g., actuators) are associated with places [162].

> Important

Note that interpreted nets from the definition are *safe (1-bounded)*, live and reversible Petri nets.

Definition 2.28. *A deadlock* is a situation in a Petri net $N = (P, T, F, \overrightarrow{M_0})$ such that from a reachable marking $\overrightarrow{M_s}$ there is no sequence of firing σ to obtain another reachable marking $\overrightarrow{M_r}$.

Definition 2.29. A Petri net $N = (P, T, F, \overrightarrow{M_0})$ is said to be *consistent* if there exists an initial marking (state) $\overrightarrow{M_0}$ and a firing sequence σ from $\overrightarrow{M_0}$ back to $\overrightarrow{M_0}$ such that every transition $t \in T$ occurs at least one in σ.

Definition 2.30. A Petri net $N = (P, T, F, \overrightarrow{M_0})$ is said to be *repetitive* if there exists an initial marking $\overrightarrow{M_0}$ and a firing sequence σ such that every transition $t \in T$ occurs infinitely often in σ.

Definition 2.31. *An incidence matrix* of a Petri net $N = (P, T, F, \overrightarrow{M_0})$ with $n = |P|$ places and $m = |T|$ transitions is an $A_{m \times n = [a_{ij}]}$ matrix (where m refers to rows, and n refers to columns) of integers, given by:

$$a_{ij} = \begin{cases} -1, & (p_j, t_i) \in F \\ 1, & (t_i, p_j) \in F \\ 0, & \texttt{otherwise} \end{cases} . \tag{2.15}$$

Example 2.8

One of formal Petri net expressions is an incidence matrix. The incidence matrix of a multi-robot example N_1 is presented in Equation 2.16.

$$A_1 = \begin{array}{c} \begin{matrix} p_1 & p_2 & p_3 & p_4 & p_5 & p_6 & p_7 & p_8 & p_9 \end{matrix} \\ \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & -1 & -1 & 1 \\ 1 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 1 & 0 & -1 & 1 & 0 & 0 \end{bmatrix} \end{array} \begin{matrix} t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \\ t_6 \end{matrix} \tag{2.16}$$

The incidence matrix A_1 consists of nine columns representing places. Similarly, transitions are represented by six rows. For instance, the cell $a_{23} = 1$ of A_1 determines that p_3 is an output place of transition t_2. Adequately, cell $a_{57} = -1$ represents input to transition t_5 from the place p_7. No connection between a place and a transition is indicated as 0 for example, $a_{25} = 0$. It is easy to notice that a graphical structure of the Petri net can be easily represented by this matrix algebraic notation.

Definition 2.32. *A place invariant (p-invariant)* of a Petri net $N = (P, T, F, \overrightarrow{M_0})$ is a vector

$$\vec{x} \geq 0 \text{ such that } A\vec{x} = \vec{0}, \tag{2.17}$$

where A is the incidence matrix of the net.

Example 2.9

Let us demonstrate an example to clarify what a place invariant stands for. The Petri net N_2 shown in Fig. 2.4 represents a dance ballroom.

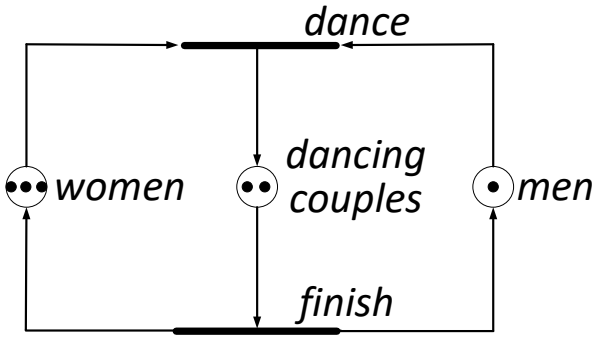


Figure 2.4: Place invariant idea

The Petri net N_2 specifies that there are men and women waiting for their dance and dancing couples in the ballroom. Firing the transition *dance* takes one waiting man and one waiting woman and puts them together on the dance floor, whereas the transition *finish* splits them again. Obviously, firing transitions (*dance* or *finish*) cannot change the number of persons in the ballroom. Since we can formulate an equation: $1 \cdot M(men) + 1 \cdot$

$M(women) + 2 \cdot M(couples) = \texttt{const}$. For initial marking $\overrightarrow{M_0}$ in the example N_2, we have a formula: $1 \cdot 1 + 1 \cdot 3 + 2 \cdot 2 = 8$. Note that a nonnegative weight w was assigned to each place $p \in P_2$ such that the weighted token sum remains constant in every reachable state. It is *a place invariant* because firing of any transition does not change the weighted token sum.

Definition 2.33. *A transition invariant (t-invariant)* of a Petri net $N = (P, T, F, \overrightarrow{M_0})$ is a vector

$$\vec{y} \geq 0 \ \texttt{such that} \ A^{\mathsf{T}}\vec{y} = \vec{0}, \tag{2.18}$$

where A^{T} is the transposed incidence matrix A of the net.

Example 2.10

The same Petri net example N_2 can illustrate a transition invariant idea. We can formulate a t-invariant when firing transitions in any order results in the same marking as before the series of firings. For instance, five firings of transitions *dance* and *finish* returns to the same marking: $5 \cdot dance + 5 \cdot finish = \texttt{const}$. Assigning to each transition $t \in T_2$ a nonnegative weight w such that firing each transition w-times does not change the state is *a transition invariant*. Transition invariant corresponds to a cycle in the reachability graph.

Theorem 2.1. A Petri net $N = (P, T, F, \overrightarrow{M_0})$ is *bounded* if it is covered by place invariants [127]. A Petri net is covered by place invariants if every place $p \in P$ belongs to at least one set of nonzero entries of a place invariant.

2.6 Classes of Petri Nets

Wide application of Petri nets, their popularity and good theoretical background encourage scientists to research their new types and classes. We present classes of Petri nets in Fig. 2.5. The more specific the class, the less complex its analysis, and the more general the class the better expressiveness it provides. We discuss the problem in detail in Chapter 3, which describes properties of Petri nets and methods of their analysis.

2.6.1 State Machines

Below are formal definitions related to state machines.

Definition 2.34. *A state machine (SM-net)* is a Petri net $N = (P, T, F, \overrightarrow{M_0})$ whose transitions $t \in T$ have exactly one input place $p_i \in P$ and one output place $p_o \in P$, i.e. $\forall t \in T : |\bullet t| = |t \bullet| = 1$.

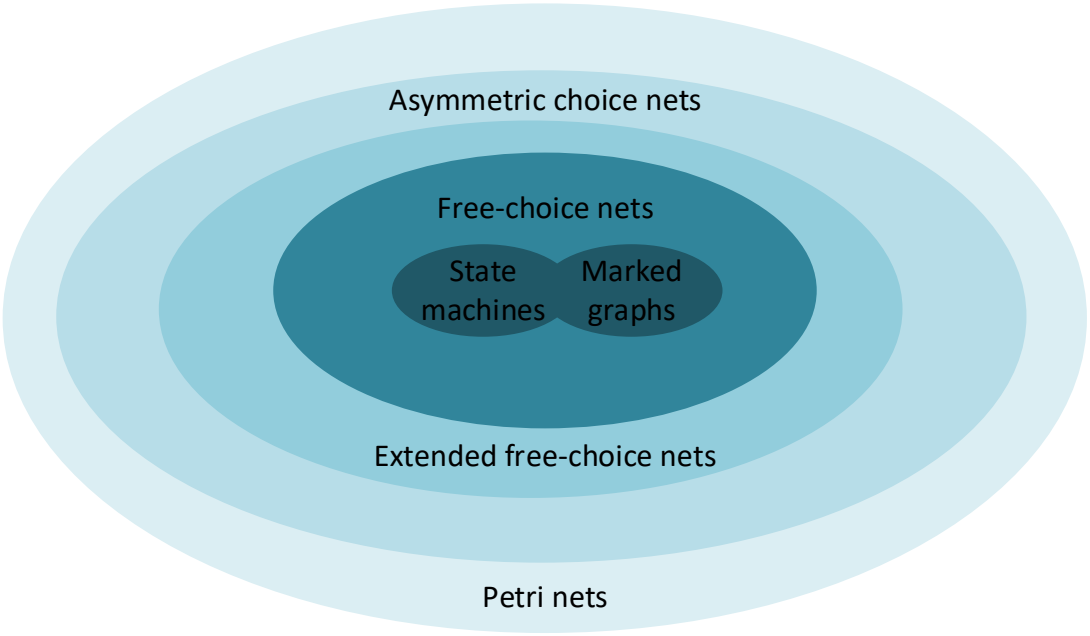


Figure 2.5: Classes of Petri nets

Definition 2.35. *A state machine component (SM-component, S-component, SMC)* of a Petri net $N = (P, T, F, \overrightarrow{M_0})$ is its subnet

$$S = (P', T', F', \overrightarrow{M_0'}), \tag{2.19}$$

such that S is an SM-net and contains exactly one token in the initial marking.

Definition 2.36. *A state machine cover (SMC-cover)* of a Petri net $N = (P, T, F, \overrightarrow{M_0})$ is a set of state machine components

$$\mathbb{S} = \{S_1, \ldots, S_n\}, \tag{2.20}$$

such that each place $p \in P$ is a place of at least one S-component $S \in \mathbb{S}$.

In other words, a state machine is such a Petri net that has just one input and one output place for any transition.

Example 2.11

See Fig. 2.6 as an example of an SM-net. Each transition $t \in \{t_1, t_2, t_3, t_4\}$ has exactly one input and one output place. We can observe that state machines have a sequential structure, and their decision behavior can be modeled. Note that there are two alternate firing sequences in the SM-net presented in Fig. 2.6: $\sigma_1 = \{t_1, t_3\}$ or $\sigma_2 = \{t_2, t_4\}$. Since every transition can have just one input and one output place, no concurrency can be specified.

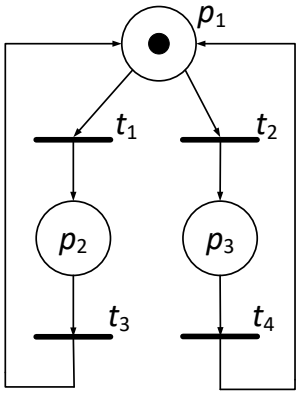


Figure 2.6: An example of a state machine

2.6.2 Marked Graphs

Below is a formal description of marked graphs.

Definition 2.37. *A marked graph (MG)* is a Petri net $N = (P, T, F, \overrightarrow{M_0})$ such that for all places $p \in P$ holds $|\bullet p| = |p \bullet| = 1$.

While state machines have one input and one output place of any transitions, marked graphs have exactly one input and one output transition for each place.

Example 2.12

An exemplary marked graph is shown in Fig. 2.7 since every place has exactly one input and output transition. Contrary to state machines, marked graphs have a parallel nature and cannot be used to model decision behavior. In the example below, places p_2 and p_3 are concurrent to each other.

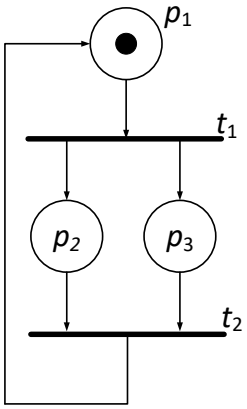


Figure 2.7: An example of a marked graph

2.6.3 Other Classes

Below are remaining definitions and notations that are necessary to explain the idea of free-choice nets, extended free-choice nets and asymmetric choice nets.

Definition 2.38. *A free-choice net (FCN)* is a Petri net $N = (P, T, F, \overrightarrow{M_0})$ such that for all places $p \in P$ is true that $|p\bullet| \leq 1 \vee \bullet p(p\bullet) = p$.

Definition 2.39. *An extended free-choice net (EFC)* is a Petri net $N = (P, T, F, \overrightarrow{M_0})$ such that for all places $p_1, p_2 \in P$, if $p_1\bullet \cap p_2\bullet \neq \emptyset \implies |p_1\bullet| = |p_2\bullet| = 1$.

Definition 2.40. *An asymmetric choice net (ACN)* is a Petri net $N = (P, T, F, \overrightarrow{M_0})$ such that for all places $p_1, p_2 \in P$ holds $p_1\bullet \cap p_2\bullet \neq \emptyset \implies |p_1\bullet| \supseteq |p_2\bullet| \vee p_1\bullet \subseteq p_2\bullet$.

2.7 Reduced Row Echelon Forms

Below are definitions and formal notations explaining the core idea of applying reduced row echelon forms in calculating invariants of Petri nets.

Definition 2.41. A matrix B is said to be in a reduced row echelon form if and only if it satisfies the following conditions [172]:

- if a row of B does not consist entirely of zeros, the first nonzero number in the row is 1 (called a *leading one*),
- any rows which consist entirely of zeros are grouped together at the bottom of B,
- in any two successive non-zero rows of B, the leading one in the lower row occurs farther to the right than the leading one in the higher row,
- each column of B that contains a leading one has zeros everywhere else.

One successful and simple technique for solving linear systems $A\vec{x} = \vec{0}$ is to reduce the coefficient matrix A of the system to a reduced row echelon form matrix B [172]. This can be accomplished by applying a sequence of elementary row operations like Gauss-Jordan elimination [172]. We know that if A is an $m \times n$ matrix and $\vec{x}$ is an n-vector then the systems $A\vec{x} = \vec{0}$ and $B\vec{x} = \vec{0}$ have exactly the same solution set. However, the benefit of such an approach is that the solution set might be read off directly from the matrix B.

There are three elementary row operations to convert any matrix A to its reduced row echelon form matrix B:

- swap two rows,
- multiply a row by any non-zero constant,
- add a scalar multiple of one row to any other row.

The above operations are used in any order to receive a matrix according to Def. 2.41.

Example 2.13

Consider a simple exemplary linear system given in Equation 2.21.

$$\begin{cases} 5x + 10y - 100 &= 0 \\ 40x - 80y &= 0 \\ 10x + 20y - 200 &= 0 \end{cases} \tag{2.21}$$

And its coefficient matrix in Equation 2.22.

$$A = \begin{bmatrix} 5 & 10 & 100 \\ 40 & -80 & 0 \\ 10 & 20 & 200 \end{bmatrix}. \tag{2.22}$$

Let us now try to solve this linear system in Equation 2.21. Note that one of three equations does not contribute any information to the linear system. The third equation is simply the first one multiplied by two. We can remove the first or third equation without any impact on the solution set.

$$\begin{cases} 5x + 10y - 100 &= 0 \\ 40x - 80y &= 0 \end{cases} \tag{2.23}$$

The linear system in Equation 2.23 can be solved by any techniques taught in high school. After the solving, we know that $x = 10$ and $y = 5$. Now, the reduced row echelon form of matrix A is presented in Equation 2.24.

$$B = \begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \end{bmatrix} \tag{2.24}$$

For this, we can form a linear system from matrix B:

$$\begin{cases} x + 0 &= 10 \\ 0 + y &= 5 \\ 0 + 0 &= 0 \end{cases} \tag{2.25}$$

The solution can be directly read off: $x = 10$ and $y = 5$. Moreover, a row with all zero entries indicates that one equation in the linear system in Equation 2.21 was redundant.

Subsequently, let us use the discussed methodology for invariant computation. As a place invariant is a solution of a linear system $A\vec{x} = \vec{0}$, the presented approach is adopted for place invariant computation in Petri nets.

Example 2.14

For example, consider an incidence matrix A of Petri net N_3 [2]. The Petri net is shown in Fig. 2.8 and its matrix B in a reduced row echelon form accomplished after Gauss-Jordan elimination in Equation 2.27.

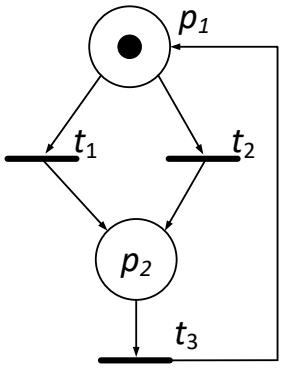


Figure 2.8: Petri net N_3

$$A = \begin{matrix} \\ -1 \\ -1 \\ 1 \end{matrix} \begin{matrix} \begin{matrix} p_1 & p_2 \end{matrix} \\ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \end{matrix}, \tag{2.26}$$

$$B = \begin{matrix} \\ 1 \\ 0 \\ 0 \end{matrix} \begin{matrix} \begin{matrix} p_1 & p_2 \end{matrix} \\ \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \end{matrix}. \tag{2.27}$$

The solution set can be easily read off:

$$\begin{cases} p_1 + (-p_2) = 0 \\ 0 = 0 \\ 0 = 0 \end{cases} \Leftrightarrow p_1 = p_2\,. \tag{2.28}$$

Note that there are just two $\vec{x} \geq 0$ integer vectors that do not include other vectors (we call it minimal) and satisfy the system of linear equations $B\vec{x} = \vec{0}$ and hence $A\vec{x} = \vec{0}$:

$$\begin{aligned} &\vec{x} = [0\ 0]^{\mathsf{T}} \text{ or} \\ &\vec{x} = [1\ 1]^{\mathsf{T}}\,. \end{aligned} \tag{2.29}$$

[2] Available online: http://www.hippo.issi.uz.zgora.pl/index.php?id=petri_net&nr=420 (accessed on December 11, 2022)

We will not consider *a trivial solution* $\vec{x} = \vec{0}$ as it belongs to a solution set of any homogeneous systems $A\vec{x} = \vec{0}$, where A is any matrix. Vector $\vec{x} = [1\ 1]$ is a place invariant as $A\vec{x} = \vec{0}, \vec{x} \geq 0$. We shall see that *some* invariants can be easily read off from the matrix in the reduced row echelon form.

2.8 Conclusions

We have briefly discussed the base of the Petri net theory supported with simple examples. The most common properties, i.e. boundedness and safeness, have been defined. A Petri net-based specification can be used as a modeling language. Owing to parallel nature of nets, the presented Petri net-based methodology is tailored to designing various concurrent control systems. The solution is strongly focused on the model verification aspect. The mathematical structure of Petri nets allows the use of a graph and linear algebra theory for system properties analysis. It is worth noting that other solutions e.g., UML diagrams do not provide such strict verification tools. It is true that UML state machines benefit from their intuitiveness and common awareness. However, Petri nets provide better model checking and conversion between notations. Moreover, structural conversions enhance Petri nets as a modeling choice, which we will discuss later. On the other hand, exponential time and relatively slow analysis algorithms are the main limitations of the proposed Petri nets. The problem will be discussed in the next chapter.

Chapter 3

Related Work

This chapter provides an in-depth overview of related work. Boundedness and safeness are at the core of various methodologies using the Petri nets theory. The problems of analyzing such properties as boundedness and safeness of Petri nets were undertaken over 30 years ago [149] and are still subject to research. See: [57, 71, 72, 83, 157, 160, 166–169] for publications from the last three years. Boundedness is an important feature of various systems, such as manufacturing systems presented in [83, 115]. A bounded Petri net determines a finite number of reachable states.

Let us imagine a control system specified by a Petri net which is in charge of an elevator. If the Petri net was unbounded, the elevator could behave in an indefinite way and enter states not foreseen by the designer. For instance, the elevator could stop between floors or go higher than it is allowed and thus cause a mechanical failure. Moreover, a safe Petri net may be used to model systems aimed at hardware implementation in microcontrollers or FPGAs — logical control (token/ no token) [155, 157]. In the elevator example, such logical behavior could make the elevator go up or could disable going up.

The most popular analysis techniques that permit for checking boundedness and safeness of a Petri net can be generally divided into two main groups [16]: based upon reachability graphs and upon structural analyses using linear algebra methodology. The first approach is based on exploration of reachability graphs. While constructing a state space, each subsequent possible reachable state is placed on the graph. In general, the number of reachable states of a system can grow exponentially (a state explosion problem). It translates into exponential run-time of algorithms analyzing properties on reachability graphs. Therefore, for some Petri nets consisting of up to several dozen places or transitions, the methods may not produce a result in the assumed time, i.e., they are not efficient in such cases. The second approach is based on a linear algebra technique. In this approach, all minimal invariants of a Petri net are computed (minimal meaning that they

are not contained in other invariants). Place invariants are used by linear algebra-based procedures to check boundedness and safeness. However, the number of all minimal invariants can also grow exponentially. Therefore, both verification paths have serious limitations because the number of reachable states and place invariants in the graph can be exponential [85, 112, 155]. This makes the computational complexity of algorithms investigating these properties exponential, which means that a solution may not be found within the assumed reasonable time.

Below, problems of reachability graph building, computing invariants, as well as analyses which attempt to handle these problems by means of popular methods of boundedness and safeness are presented. Additionally, computer-aided tools used in concurrent control specification based on Petri nets are mentioned.

3.1 Reachability Graphs

Finite graphs in which every reachable marking is explicitly represented by a node can be prepared [128]. Such graphs are known as reachability graphs [112]. Petri net properties can be directly obtained from exploration of such graphs. Such analyses can guarantee that a Petri net is bounded or unbounded, safe or unsafe, or can reveal if it contains any deadlocks. However, this approach can be inefficient as the number of reachable states can be exponential. It means that the operation of the algorithm may not be finished within the assumed time. Net reduction, i.e. simplification of a Petri net with preservation of its boundedness, safeness, and other behavior, can be one of practical solutions to this problem [155]. Nevertheless, such reduced graphs can be still too large to be computed [53, 155]. The size of reductions depends on the characteristics of a Petri net. Some Petri nets can be significantly reduced to make it possible to analyze them efficiently. On the other hand, other Petri nets may reduce marginally or not at all, which makes it impossible to verify them.

3.1.1 Overview

As proved by Lipton in [101], the building of a reachability graph requires exponential space and decidable marking of its reachability problem [110]. Although it is proved to be space-hard for Petri nets in general [101], for some classes of Petri nets e.g., acyclic nets, the state reachability problem can be resolved by means of an integer linear programming problem. Moreover, it can be solved in polynomial time in state machines and marked graphs. The reachability problem is part of the Petri net theory. Many properties including boundedness and safeness can be resolved by reachability analysis of a system.

Below, the most interesting properties will be described in order to move on to construction techniques of reachability graphs. Binary decision diagrams (BDD) can be used for some checking properties by the symbolic traverse technique. Nevertheless, since

BDD data structure is based on place invariants, reconstruction of the behavior of a Petri net is not straightforward [107].

Valmari's set of methods [150] is perfectly applicable to Petri nets with identical concurrent structures. For instance, various identical workflows are triggered simultaneously, and they run in parallel. However, the given requirement that all workflows have identical structure, which is a very restrictive presumption, limits the applicability of this approach. A subnet reducing into a single transition with preserving properties like boundedness is presented in papers [48] and [137]. In [90], the author creates a dependency graph for removing some substructures but without changing the property of reachability. However, the limitation of these approaches is that they highly base on some particular substructures of Petri nets.

Recently, a state compression approach [24, 60] has been proposed by Cabasinoet et. al. The benefit of this methodology is that only a part of reachability space (called basis markings) is enumerated. This technique can be effectively used to solve controllability [106] or opacity problems [145, 146] in Petri nets. Moreover, numerous different methods with new classes of Petri nets are developed to handle the marking reachability problem. For example, siphon analysis is applied to determine if deadlock markings exist in S^3PR nets [54, 98, 99].

In paper [107], a practically efficient algorithm based on a reachability graph solves the marking reachability problem in Petri nets. That method has wide relevance, since it does not depend on particular substructures of Petri nets. Reachability sets can be precisely characterized by this technique.

There are available tools which implement state space generation to determine the properties of a Petri net. There is a great deal of computer-aided design software to support the specification of Petri nets. Up to one hundred such programs can be identified. Many of them are solutions presented in the 1990s, e.g., PAPETRI, Xpetri, PROD, INA, SPNP and others. Websites of those programs do not respond, and probably their development was completed many years ago. Another group are programs whose last version was created in the years 2000–2010, among which we can distinguish such programs as: Yasper [75] without the option of analyzing Petri nets, JSARP [114] with analysis of only boundedness and liveness on reachability graphs, JARP with a state space generator, but without information on the properties. The most popular and cited in recent publications with a wide range of verification are IOPT-Tools [26, 63, 65–67, 117–120, 157] and PIPE [23, 51, 89]. Both CAD programs perform a check of properties such as boundedness or safeness based on reachability graphs. Due to the state explosion problem, some Petri nets with several or more places and transitions cannot be checked in the assumed time.

3.1.2 Construction

The construction of a reachability graph that contains all the reachable markings of a Petri net is shown in the Algorithm 3.1. This foundational method proposed by Murata in [112] is claimed to be a reference reachability graph determining algorithm. Paper [112] also introduces the symbol ω as an indicator of a place that accumulates tokens infinitely in order to maintain a finite reachability graph for unbounded Petri nets.

Data: A Petri net $N = (P, T, F, \vec{M_0})$
Result: The reachability graph $R = (\mathbb{M}, \Sigma)$
1 $\vec{M} \leftarrow$ an initial marking $\vec{M_0}$;
2 **while** *exists a new marking that can be reachable from* $\vec{M}$ **do**
3 **foreach** *enabled transition* t *at* $\vec{M}$ **do**
4 $\vec{M_s} \leftarrow$ a marking after t fired from $\vec{M}$;
5 **if** $\vec{M_s} \notin \mathbb{M}$ **then**
6 **if** *exists* $\vec{M_r}$ *on the path from* $\vec{M_0}$ *that* $\forall p \in P, M_s(p) \geq M_r(p)$ **then**
7 **foreach** $p \in P, M_s(p) > M_r(p)$ **do**
8 $M_s(p) \leftarrow \omega$;
9 **end**
10 **end**
11 $\mathbb{M} \leftarrow \mathbb{M} \cup \{\vec{M_s}\}$;
12 $\Sigma \leftarrow \Sigma \cup \{t\}$;
13 **end**
14 **end**
15 **end**

Algorithm 3.1: Construction of a reachability graph

Building a reachability graph begins with initial marking as a current state. In turn, it is checked whether it is possible to reach a new state from this marking, i.e. if there are enabled transitions. Then, each subsequent enabled transition is fired, and a new state is generated. The new state becomes a current marking and further new states are searched for according to the same procedure. After each new marking, if such a state $\overrightarrow{M_r}$ is obtained on the path from the initial marking $\overrightarrow{M_0}$ to the current marking $\overrightarrow{M_s}$ that $\forall p \in P, M_s(p) \geq M_r(p)$, then a symbol ω is placed for $p \in P, M_s(p) > M_r(p)$, indicating that for those places tokens will accumulate in an infinite way.

Example 3.1

The reachability graph shown in Fig. 2.3 of the Petri net N_1 was obtained after performing the following steps described in Algorithm 3.1. The first state is the initial marking M_0. We then check whether it is possible to gain a new state by seeking the enabled transitions from the current state M_0. In the case of the M_0 state, two transitions are enabled to fire: t_1 and t_4. Therefore, each enabled transition is fired in turn and a new state is determined. In our example, after firing t_1, we get marking M_1, and after transition t_4,

marking M_2. For each established state, we check the unbounded place condition (line 6 of Algorithm 3.1). For the M_1 and M_2 states, all places are bounded — condition does not apply. In the same way, subsequent states are determined until no transition is enabled or a new state cannot be resolved (all of them have been placed in the graph).

3.2 Invariants

Computation of invariants is one of the most popular techniques of verification and analysis of Petri nets [95, 109, 112, 155]. Place invariants can be used to analyze important properties of Petri nets, such as boundedness and safeness [109, 112, 154, 155, 157, 165, 168]. It should be emphasized that checking the properties of Petri nets by obtaining invariants is usually more efficient than exploring the reachability graph, especially for larger nets, i.e., with a dozen places or transitions and more.

Analysis of boundedness requires computation of a place invariant cover of the Petri net [127, 168]. Reisig [127] proves that place invariant covered Petri nets are *bounded* and their transition invariant is repetitive. Repetitiveness can be used for the investigation of significant properties, such liveness or absence of deadlocks. Also, according to [112], each t-invariant of a Petri net is p-invariant of a dual net. Notice that a dual net has a transposed incidence matrix. A Petri net is said to be covered by place or transition invariants if its every place or transition belongs to at least one set of corresponding non-zero entries in a p- or t-invariant, respectively.

The most trivial approach to find minimal place invariants $\vec{x} \geq 0$ that $A\vec{x} = \vec{0}$ is testing all $\vec{x}$ permutations in domain $x(i) \in \{0, 1\}$, where $i \in \{1..n\}$. Similarly, minimal transition invariants can be found when solving equation: $A^{\mathrm{T}}\vec{y} = \vec{0} \Leftrightarrow \vec{y}A = \vec{0}$. By minimal invariants we mean those that contain no other invariants. Each invariant can be multiplied by an integer to get another invariant. Thus, whenever we search for invariants, we refer to the minimal ones, i.e. those that are not contained in others. Algorithm 3.2 shows a trivial procedure to obtain place invariants for any Petri net.

```
   Data: A Petri net N = (P, T, F, M_0)
1  A ← an incidence matrix of N;
2  C ← Find All Binary Permutations (|P|);
3  foreach vector x of matrix C do
4  |  if A × x = 0 then
5  |  |  print "vector x is the p-invariant of net N";
6  |  end
7  end
```

Algorithm 3.2: Place invariants calculating a trivial procedure

This algorithm is obviously noneffective because we need $2^{|P|}$ equality check operations. Assuming one matrix multiplication takes 10^{-4} seconds [1], for a middle-size Petri net (e.g., $|P| = 30$) the whole procedure lasts approx. 30 hours. The application of this method for a 43-place Petri net requires as much time as the age of the author of this dissertation.

In general, the linear algebra approach to obtain place invariants can be treated as *a Boolean satisfiability problem*. The problem is represented as $p_1 + p_2 + \cdots + p_n = 0$, where $p_i, i \in \{1..n\}$ is a Boolean variable, and thus is *NP-complete* [38]. In [38], Cook proved that any problem in *NP class* can be reduced in polynomial time by *a deterministic Turing machine*. An important corollary of the described theorem is that *Boolean satisfiability* is equivalent to the *P versus NP problem*, which is widely considered the most important unsolved problem in theoretical computer science [86].

The first invariant computing method is proposed in [144]. All invariants are produced one by one. The algorithm can be stopped at any time of its execution. While polynomial amount of memory is used, the computation time in the worst case is exponential.

The Fourier-Morzkin method [109] is another well-known proposition for computing all invariants. It has the following critical drawbacks. Even if invariants exist, none of them may be computed because of an overflow caused by candidate vectors for invariants whose number may become exponential or many non-minimal invariants may be included too. There are several ideas to improve the Fourier-Morzkin method, which have been suggested in some papers [36, 109, 142, 143]. As for this approach, only one or a few minimal invariants can be found instead of all. It may shorten computation time and save memory.

A relatively new algorithm GMST is presented in [141]. It makes it possible to compute at least one invariant in short computation time under condition that any invariant exists in an inputted Petri net. Schmidt [130] worked out a sketch of an algorithm to compute invariants of algebraic nets. His method has a number of disadvantages, including intolerable time complexity and computational problems [130].

A Toudic method algorithm [147] features means of matrix transformations for obtaining non-negative integer solutions (invariants). However, the complexity of this solution is exponential, which restricts the analysis of real-life models [175]. Zaitsev implements this algorithm in [174] and basing on the decomposition of Petri nets, improves it in [175], to reach exponential acceleration with respect to the number of places. The same author summarizes in 2006 [173] that when investigating properties of Petri nets, all well-known methods display exponential computational complexity.

Finally, the most popular and commonly used algorithm for computation of invariants was initially introduced in [109]. The Martinez-Silva method is widely known and described in many papers [154, 165, 168]. It also constitutes a basis for other methods of

[1] Octave software on AMD Phenom II X4 965 processor (3.4 GHz x 4) and 12 GB of RAM

computing invariants or improving this reference algorithm. The procedure is presented step by step in Algorithm 3.3.

Data: A Petri net $N = (P, T, F, \vec{M}_0)$
Result: Rows of D that correspond to the found p-invariants
1 $D \leftarrow$ an identity matrix of size $|P|$;
2 $A \leftarrow$ an incidence matrix of N;
3 $A^{\mathsf{T}} \leftarrow$ `Transpose Matrix` (A);
4 $Q \leftarrow$ union matrix of D and A^{T};
5 **foreach** *column j of matrix A^T* **do**
6 find row pairs that their sum is equal to 0 and add it to Q;
7 delete rows that intersection with j-th column is not equal to 0;
8 reduce redundant rows of Q;
9 **end**

Algorithm 3.3: Martinez-Silva's algorithm for calculating place invariants

Algorithm 3.3 is based on algebra (matrix operations). Firstly, a unit matrix Q is formed. This matrix conjugates two matrices: an identity matrix D of places number size of a Petri net and a transposed incidence matrix A^{T} of the net. Note that matrix D will hold the obtained place invariants after the execution of the algorithm. Subsequently, for each column of the transposed incidence matrix, the algorithm searches for row pairs that annul the adequate column. Those rows are appended to matrix Q. Then non-minimal invariants, being supersets, are removed from Q. Finally, rows of A^{T} that contain all zeroes are related to the proper place invariant in matrix D.

However, exponential computational complexity is the main limitation in the analysis of invariants by this method and its derivatives, since it makes it possible to obtain all the minimal invariants of a Petri net [155]. Moreover, place invariants are presented after the algorithm terminates its operation.

3.3 Property of Boundedness

Apart from safeness, this property is the most common subject matter in surveys on Petri nets. A bounded Petri net has a finite number of tokens for each place in any reachable state. In turn, the number of tokens is nondeterministic in Petri nets that are unbounded.

Example 3.2

Figure 3.1 shows the Petri net N_4 [2] given as an example of an unbounded net. We can see that in the firing loop sequence $\sigma = \{t_1, t_2, t_1, t_2, t_1, t_2, \cdots\}$, the new tokens are produced in place p_3 ad infinitum.

[2] Available online: `http://www.hippo.issi.uz.zgora.pl/index.php?id=petri_net&nr=290` (accessed on December 11, 2022)

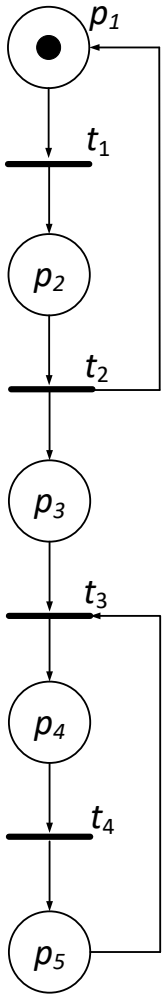


Figure 3.1: An example of an unbounded Petri net

If a Petri net is unbounded, the graph representation grows infinitely large [112]. Unbounded Petri nets are a wide topic that requires different analytic tools, so it is out of the scope in our discussion. In [104], authors address this subject and propose complex reachability trees to analyze such nets. Generally, such unbounded nets have little practical application in cases studied within this dissertation.

Example 3.3

Fig. 3.2 shows a bounded Petri net N_5. The idea of this example is based on the famous video game series — The Settlers [3]. In the Petri net with $|P_5| = 5$ places and $|T_5| = 3$ transitions, we can see ten tokens in the place p_1 (*population*) representing ten free settlers. There are also five available *axes* (p_2), three *iron* (p_4) bars and seven units of *coal* (p_5). During the game (when changing the states), if a player needs to *recruit* (fired t_2) a *woodcutter*

[3] More information available online: `https://en.wikipedia.org/wiki/The_Settlers` (accessed on December 11, 2022)

(p_3), we have to provide one free settler from the population (p_1) with one axe (p_2). When the woodcutter (p_3) is *retired* (t_1), he comes back to the free population (p_1) and returns his axe (p_2). At the same time, a new axe can be made by *Toolsmith's Works* (t_3) from iron (p_4) and coal (p_5).

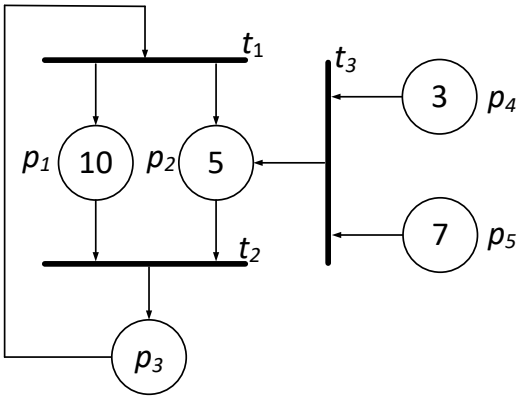


Figure 3.2: An example of a bounded Petri net

Table 3.1 presents minimal and maximal numbers of objects (axes, iron, coal, etc.) in any time of the game. Note that the number of objects is always (in any reachable state) *bounded*. In other words, the object neither disappears, nor is it preternaturally multiplied.

Table 3.1: Minimal and maximal bounds of the Petri net N_5

	p_1 (population)	p_2 (axe)	p_3 (woodcutter)	p_4 (iron)	p_5 (coal)
min bounds	2	0	0	4	0
max bounds	10	8	8	7	3

Algorithm 3.4 presents boundedness analysis by means of exploration of reachability graphs. The function *Construct a Reachability Graph* is described in Algorithm 3.1. It should be remembered that for some Petri nets this approach to the construction of graphs cannot be completed in reasonable time because of computational complexity.

Boundedness analysis with the application of Martinez-Silva invariants computation method is shown in Algorithm 3.5. Obtaining place invariants is proceeded by the function *Calculate Place Invariants* detailed in Algorithm 3.3. The whole procedure essentially checks the coverage of a Petri net by the obtained place invariants. If a Petri net is covered then it is bounded.

Below, application and usefulness of bounded Petri nets in recent research and in various proposed solutions is presented. Kaid et al. proposes to apply Petri nets to the scheduling and deadlock analysis in reconfigurable manufacturing systems [83]. The

```
   Data: A Petri net N = (P, T, F, M⃗_0)
   Result: The net N is bounded or unbounded
 1 R = (𝕄, Σ) ← Construct a Reachability Graph (N);
 2 foreach state M⃗ at 𝕄 do
 3 |   foreach place p at M⃗ do
 4 |   |   if M(p) = ω then
 5 |   |   |   is_bounded ← false;
 6 |   |   end
 7 |   end
 8 end
 9 if is_bounded then
10 |   print "net N is bounded";
11 else
12 |   print "net N is unbounded";
13 end
```

Algorithm 3.4: Boundedness analysis based on a reachability graph

algorithm uses the most important properties of Petri nets, including boundedness. The author's technique permits dynamic modification of the structure of the manufacturing system without affecting its behavioral properties (boundedness, liveness, reversibility).

```
   Data: A Petri net N = (P, T, F, M⃗_0)
   Result: The net N is bounded or uncovered
 1 I ← Calculate Place Invariants (N);
 2 foreach column p of matrix I do
 3 |   if p = 0 then
 4 |   |   is_covered ← false;
 5 |   end
 6 end
 7 if is_covered then
 8 |   print "net N is bounded";
 9 else
10 |   print "net N is uncovered by place invariants";
11 end
```

Algorithm 3.5: Boundedness analysis based on place invariants cover

The solution presented in [33] proposes deadlock recovery policy in flexible manufacturing systems. Generally, the proposed algorithm performs building reachability graphs of Petri nets to meet such specification requirements as reachability or liveness. The task requires verification of boundedness for the designed concurrent control system.

A system specified by a bounded Petri net is necessary for the diagnosability analysis technique described in [125]. The paper also involves verification of reachability graphs of Petri nets. In order to reduce the computational complexity due to the state explosion problem, a special class of Petri nets known as "modified verifier nets" is introduced. An

automaton "verifier" makes it possible to detect malfunctions of the modeled system.

Bounded Petri nets are applied in [14] for a synthesis algorithm. The system is composed from smaller sub-components — transition systems. A symbolic representation of states is involved to generate such components to be synthesized. The authors experimentally verified their algorithm and implemented it as a tool. It was shown that application of bounded Petri nets resulted in significant reduction in the synthesis.

Decomposition methods of persistent Petri nets are introduced in [21]. The paper states that bounded and reversible nets can be decomposed into smaller, transition-disjoint cycles. The authors propose several methods based on reachability graphs and linear algebra techniques. Boundedness of Petri nets is a crucial property and is required in such analyses.

The property of boundedness finds its application in an algorithm analyzing so-called "non-blockingness" of a specified system. This property is especially often utilized in the supervisory control theory of discrete-event systems [72]. The proposed method is meant to reduce analytical complexity by avoiding the space explosion problem. This is accomplished with integer linear algebra as opposed to constructing a traditional reachability graph.

Concluding the above, it can be observed that the property of boundedness is essential in design, analysis, verification, and also decomposition of Petri net-based concurrent control systems. Most practices require a bounded Petri net as their input data for further processing. Moreover, bounded Petri nets find their application when some kind of memory is necessary, contrary to safe nets which fit into e.g., logical control.

3.4 Property of Safeness

Safeness can be treated as stronger boundedness. Safe Petri nets are also called 1-bounded, because the maximal number of tokens for each place is one in any reachable state. Places in safe Petri nets display binary behavior: there is one token in a place or there is no token in a place for any reachable marking. Because of this feature, safe Petri nets find their application in interpreted nets that have additional sets of logical input/ output signals.

Multiple publications all over the world feature the research of safeness [155]. The basic theory regarding safety analysis of Petri nets is introduced in [37, 46, 155]. Zaitsev [173] provides a necessary and sufficient condition for the property of safeness

$$\exists \vec{x} > 0, A\vec{x} = \vec{0}. \tag{3.1}$$

Like boundedness, safeness can be tested on reachability graphs, as presented in Algorithm 3.6. The advantages and limitations of the solution are the same as in the case of analysis of boundedness.

Moreover, the Martinez-Silva method of calculating invariants can also be adapted to the verification of safeness. Instead of covering place invariants, Algorithm 3.7 checks

```
   Data: A Petri net N = (P, T, F, M⃗_0)
   Result: The net N is safe or unsafe
 1 R = (𝕄, Σ) ← Construct a Reachability Graph (N);
 2 foreach state M⃗ at 𝕄 do
 3 |   foreach place p at M⃗ do
 4 |   |   if M(p) > 1 then
 5 |   |   |   is_safe ← false;
 6 |   |   end
 7 |   end
 8 end
 9 if is_safe then
10 |   print "net N is safe";
11 else
12 |   print "net N is unsafe";
13 end
```

Algorithm 3.6: Safeness analysis based on a reachability graph

that the resulting invariants form correct state machine components (ref. Def. 2.35), and then whether a given Petri net is covered by these S-components.

```
   Data: A Petri net N = (P, T, F, M⃗_0)
   Result: The net N is safe or uncovered
 1 I ← Calculate Place Invariants (N);
 2 foreach row x⃗ of matrix I do
 3 |   if x⃗ is not correct SMC then
 4 |   |   I ← I − {x⃗};
 5 |   end
 6 end
 7 foreach column p of matrix I do
 8 |   if p = 0 then
 9 |   |   is_SMCs_covered ← false;
10 |   end
11 end
12 if is_SMCs_covered then
13 |   print "net N is safe";
14 else
15 |   print "net N is uncovered by SMCs";
16 end
```

Algorithm 3.7: Safeness analysis based on SMCs cover

Some more advanced algorithms checking safeness are proposed and shown in [16, 19, 20, 73, 87, 155]. However, they are applicable for particular classes of Petri nets [16, 19, 20, 73, 87, 155]. For instance, in marked graphs, the net which is live is safe if and only if each place $p \in P_{MG}$ belongs to a cycle, which possesses exactly one place which is marked under $\vec{M}_{MG}$. For this class of Petri net, analysis problem is time polynomial.

A parallel algorithm implemented in the HiPS tool [79] can be used for efficient (according to authors) safeness analysis of free-choice net class only.

Silva et al. [133] show a noteworthy linear programming approach to the safeness checking problem. This property is satisfied if the following equation is bounded and its optimal value is less than or equal to $k = 1$.

$$\min\{\vec{x}^{\mathrm{T}} M_0 | \vec{x}^{\mathrm{T}} A \leq 0; \vec{x} \geq \vec{e_p}\}\,, \tag{3.2}$$

where $\vec{e_p}$ is a vector with $e_p[p_i] = 0$ for all $p_i \neq p$ and $e_p[p] = 1$. Thus, linear programming problems can be solved by well-known polynomial time algorithms [133]. Nevertheless, the property of safeness cannot be concluded if the optimal value of the linear programming equation is greater than k [133].

Several design solutions require a safe Petri net as an input to their methods, for instance, presented in [14, 27, 34, 57, 164]. The Authors of [61] propose modeling of a discrete event system based on a safe Petri net. With the theory of supervisory control, a specification which requires avoiding a set of forbidden markings is typically considered. The proposed algorithm designs a maximally permissive deadlock avoidance controller for a safe Petri net system. Furthermore, apart from safeness, other control problems are analyzed in the paper.

Based on the theory of general regions, Carmona et al. [27] present an algorithm for bounded or safe Petri net synthesis. An efficient synthesis methodology for concurrent control systems is introduced. Similar presumptions are treated in [49],i.e. a safe Petri net is obligatory as an input to construct a controller. Next, maximal permissive controllers are obtained by computation of invariants. The work introduces a systematic algorithm to depreciate the number of linear constraints corresponding to the forbidden states. This is performed by means of non-reachable states and by building constraints using a systematic scheme.

In the paper [39], labeled as Petri nets, a given net needs to be safe, while transitions of the net can have labels with symbols from a considered alphabet. Moreover, safe nets are also appropriate for verification aspects. The Authors claim that every system of finite states can be expressed as a safe-labeled Petri net, and as a proof, a novel synthesis algorithm is proposed.

Safe Petri nets for a natural and widespread model of concurrent control systems are considered in [55]. It proposes a notion of "pullback" for a Petri net and according to the Author, pullbacks can be mainly useful to design two concurrent control systems. Those controllers share some resources and are synchronized by means of common events. The paper grants a simple construction for pullbacks of safe Petri nets.

The paper [164] offers efficient concurrency and sequentiality analysis solutions committed to the control part of a cyber-physical system specified by a safe Petri net. The introduced idea is based on the hypergraph theory and combines computation of subsequent exact transversals in a c-exact hypergraph in a polynomial time. The proposed method is supported by adequate theorems, algorithms, and proofs.

Profits of specification with the application of safe Petri nets are shown in [34]. The Authors discuss analytical methods of computational complexity for three Petri net classes: acyclic nets, conflict-free nets, and free-choice nets. All the analyzed Petri nets are required to be safe. Safe Petri nets are also subject of research in [22], where a safe Petri net is converted into a colored Petri net and again into an uncolored net to prove that all the nets in this construction have the same partially ordered multisets.

The Authors in the paper [57] submitted a model verification tool called *AdamMC*. This tool is devoted to safe Petri nets that acquire a Petri net-based specification as its input in PNML (Petri net markup language) format, extended with LTL (linear-time temporal logic) forms connected to places and transitions of the Petri net. The proposed algorithms reduce computational complexity by including sequential and parallel optimization processes.

Coverability analysis of a system specified by Petri nets is proposed in [52]. The algorithm is based on utilization of a satisfiability modulo theories solver to produce an inductive invariant. The presented model checking analysis uses integer arithmetic. Furthermore, the Author discusses similar methodologies using experimental evaluation of methods.

Summarizing the above discussion, like boundedness, safeness is a principle and a relevant property in modeling and decomposition of Petri net-based concurrent control systems. Many of the presented techniques require safe Petri nets as input data in algorithms and tools for further verification and implementation.

3.5 Conclusions

The most common problems in the analysis of Petri nets, such as the reachability problem and computation of invariants, along with the properties of boundedness and safeness, have been studied in this chapter. A broad literature review, regarding in particular the application of bounded or safe Petri nets has also been presented, together with a survey of algorithms useful for verifying the properties. The chapter also provides descriptions of known algorithms for constructing reachability graphs or computing all minimal invariants. These algorithms will serve as reference methods in the experimental research presented in Chapter 5, because the Petri net-based approach can be used as a modeling language. The aim of the discussion is to design various concurrent control systems.

Chapter 4

Methods of Analyzing Boundedness and Safeness

In this chapter, novel ideas for possible improvement of Petri nets analysis methods are presented. The proposed novel algorithms are not only dedicated to larger Petri nets, where, as expounded in detail in the previous chapters, exact methods cannot compute in the assumed time. For larger Petri nets, i.e. nets with several and more places, the described algorithms fail to provide a perfect solution for all cases, because, as discussed earlier, the problems are of exponential character. However, the novel ideas presented in this chapter will certainly support concurrent control designers in the field of modeling and analyzing Petri nets.

4.1 Boundedness

In the proposed methods for boundedness, we focus on analyses based on reachability graphs as well as on a linear algebra approach. Each of these two approaches finds its application, as both have their own benefits and limitations. As discussed in the previous chapter, these problems possess exponential computational complexity in the general class of Petri nets [16, 82, 101]. Therefore, the proposed algorithms are oriented toward efficiency and effectiveness. Efficiency is understood as obtaining a response to a given method within the assumed time. And effectiveness is reflected in the correctness of the attained result. Detailed research of the proposed methods in terms of their efficiency and effectiveness is presented in Chapter 5.

4.1.1 A Method Based on Reachability Graphs

As explained in the previous chapter, analyses of boundedness can be conducted by reviewing reachability graphs. This solution has an obvious disadvantage, since it requires examining all Petri net markings, and this translates into exponential computational complexity. Even for relatively small Petri nets, the current tools for testing boundedness based on state space analysis may not yield results within the assumed time, i.e., they are not efficient. Some popular computer aided tools, together with their limitations, were discussed in Chapter 3. While constructing a graph of the reachable states of a given Petri net according to the reference algorithm shown in Algorithm 3.1, we can see that if the conditions in Equation 4.1 are met, a symbol ω is inserted into a place that will accumulate tokens indefinitely.

$$\begin{aligned} &\exists \overrightarrow{M_r} \text{ on the path from } \overrightarrow{M_0} \text{ that} \\ &\qquad\qquad \forall p \in P, M_s(p) \geq M_r(p)\,, \\ &\Longrightarrow \forall p \in P, M_s(p) > M_r(p) \colon M_s(p) \leftarrow \omega \end{aligned} \tag{4.1}$$

where $\overrightarrow{M_r} \neq \overrightarrow{M_s}$ and $\overrightarrow{M_r}$ are any reachable markings on the graph from the initial state to the current analyzed marking $\overrightarrow{M_s}$. The authors used this technique to keep the graph finite even for unbounded Petri nets. We shall apply this to analyze the property of boundedness and during the construction of the graph, we shall interrupt its operation when such a place is found. It can be noticed that the proposed algorithm finds its special application for Petri nets that are unbounded, because it is not required to explore all markings of a Petri net to prove its unboundedness. The novel algorithm can work best in cases where a Petri net is suspected of being unbounded. For example, the designer has prepared a specification of a system based on a Petri net and wishes to check that he did not make any mistakes by creating an unbounded net, which would lead into an indefinite number of system states and its unpredictable behavior. The novel solution presented here will also co-exist with other methods proposed further, e.g.. a method on a reduced row matrix, which is dedicated to rapid initial classification of boundedness.

A method for determining boundedness based on the introduced idea is presented in Algorithm 4.1. The novel solution is mainly aimed at prompt denial of a Petri net being bounded. The word *prompt* suggests that there is no need to construct an entire reachability graph. The option of analyzing boundedness already at the level of building state space, and not after its completion, as it is usually in the case with other tools, is an indisputable advantage of this proposal. Common-use tools are mainly dedicated to general analysis. Thus, for example, first the state space of a Petri net is constructed and then a certain property of the net is examined, such as boundedness, safeness or other. On the other hand, the introduced solution is restricted directly to studying the property of boundedness. The construction of the reachability graph is done on lines 2–18 by means of Algorithm 3.1. If a place is found that will keep tokens infinitely (lines 6–8), it is a place where the number of tokens in the current building state $\overrightarrow{M_s}$ is greater

than the number of tokens for the same place in marking $\overrightarrow{M_r}$. And $\overrightarrow{M_r}$ is any marking from the initial state $\overrightarrow{M_0}$ to $\overrightarrow{M_s}$ (excluding) that for all places: $M_s(p) \geq M_r(p)$ is satisfied. Then, such a place is marked by ω in the marking $\overrightarrow{M_s}$ on the graph. Next, the algorithm immediately terminates the graph construction (line 10) with a result of unboundedness (line 22). Otherwise, all states are acquired, and the net is bounded. Therefore, the proposed method is aimed at potentially unbounded Petri nets and perfectly combines with other algorithms proposed in this chapter. Details of effectiveness and efficiency and their applications will be more broadly discussed in Chapter 5.

```
   Data: A Petri net N = (P, T, F, M⃗_0)
   Result: The net N is bounded or unbounded
 1 M⃗ ← an initial marking M⃗_0;
 2 while exists a new marking that can be reachable from M⃗ do
 3     foreach enabled transition t at M⃗ do
 4         M⃗_s ← a marking after t fired from M⃗;
 5         if M⃗_s ∉ 𝕄 then
 6             if exists marking M⃗_r on the path from M⃗_0 that ∀p ∈ P, M_s(p) ≥ M_r(p) then
 7                 foreach place p ∈ P that M_s(p) > M_r(p) do
 8                     M_s(p) ← ω;
 9                     is_bounded ← false;
10                     break;
11                 end
12             end
13             𝕄 ← 𝕄 ∪ {M⃗_s};
14             Σ ← Σ ∪ {t};
15             M⃗ ← M⃗_s;
16         end
17     end
18 end
19 if is_bounded then
20     print "net N is bounded";
21 else
22     print "net N is unbounded";
23 end
```

Algorithm 4.1: The proposed boundedness analysis on a reachability graph

Example 4.1

Below, Algorithm 4.1 is presented step by step on the example of the Petri net shown in Fig. 3.1. The input to the algorithm is the Petri net $N_3 = (P_3, T_3, F_3, M_0)$ and the expected result is a text output: net N_3 is unbounded. At the beginning, let $\vec{M}$ be equal to the initial state $\overrightarrow{M_0} = [1\ 0\ 0\ 0\ 0]$ (line 1). Next, there is a possibility of obtaining a new state (line 2), thus we fire all the enabled transitions in the current state one by one (line 3). At this stage, it is one enabled transition t_1, after that we obtain a new state $\overrightarrow{M_s} = \overrightarrow{M_1} = [0\ 1\ 0\ 0\ 0]$

(line 4). Then the method checks whether the condition in Equation 4.1 is true (line 6), i.e., there is such a state $\overrightarrow{M_r}$ reachable from the initial marking that: $M_s(p) \geq M_r(p)$ for all p places. No, there is no such a state at present. We add the current state to the collection of markings (line 13), and the fired transition to the set of fired transitions (line 14). Further, the algorithm searches for another state, i.e. the next iteration (line 2). The new state can be obtained by firing the transition t_2 (line 3), so that $\overrightarrow{M_s} = \overrightarrow{M_2} = [1\ 0\ 1\ 0\ 0]$. Again, the checker (line 6) inspects a state that satisfies the condition in Equation 4.1. This iteration such a state $\overrightarrow{M_r}$ exists on the path from $\overrightarrow{M_0}$ to $\overrightarrow{M_{s=2}}$. Therefore, for any place such that: $M_{s=2}(p) > M_{r=0}(p)$ an ω can be assigned (lines 7–8). The place p_3 is a place where the tokens will accumulate indefinitely since: $M_2(3) > M_0(3)$), i.e., the place is unbounded. It can be noticed that loop of firings $\{t_1, t_2\}$ holds tokens in the place p_3. Finally, the algorithm finishes searching for new states (line 10) and informs that net N_3 is not bounded (line 22) as expected.

Note that the algorithm for the Petri net N_3 in Fig. 3.1 terminates with the correct result only after two states are obtained from the initial marking. In the case of the reference method, it is necessary to build an entire reachability graph, in this example consisting of eight distinct markings. In the worst case, such as a bounded Petri net, the proposed algorithm requires the preparation of a full graph. However, as will be shown by the experimental results presented in Chapter 5, the operating time has been optimized in practical application. The initial suspicion of unboundedness of a Petri net is a principal element, for which a novel solution described in Section 4.1.3 has been prepared.

4.1.2 A Method Based on Invariants

This subsection introduces an idea for a novel technique to analyze boundedness. The proposed method is based on computation of place invariants. It has been shown that place invariant coverage guarantees boundedness of a Petri net. The reference algorithm proposed by Martinez-Silva searches for all minimal invariants. This approach was adopted by Algorithm 3.5 to check coverage and thus analyze boundedness. However, as outlined at length in Chapter 3, theoretical computational complexity is exponential, which stands that the method may not be completed within the assumed time even for relatively small nets. Detailed experimental results are discussed in Chapter 5. It can be noticed that for the analysis of boundedness, it is enough to calculate as many place invariants that cover a Petri net as possible. Therefore, the proposal uses the Martinez-Silva algorithm and modifies it to interrupt its operation once the coverage is achieved.

Firstly, we will show the key steps of the algorithm, together with their description. Subsequently, the benefits and limitations of the solution will be discussed.

The proposed Algorithm 4.2 involves the linear algebra technique. The method is

based on the Martinez-Silva computation algorithm given in Algorithm 3.3 as stated above. However, the novel solution does not require calculation of all place invariants in a Petri net. The method performs as follows. At the beginning, the unit matrix Q of matrices: identity D and transposed incidence matrix A^{T} of the Petri net is formed (lines 1–4). In the next stage, the algorithm searches for place invariants by manipulating the matrix Q (lines 7–9). The subsequent transitions of the Petri net are examined to zeros (filling all elements with zero value) matrix A^{T}. In the meantime, matrix D saves partially obtained invariants. If all entries of any row in A^{T} are equal to zero, then the correct p-invariant can be formed from matrix D (lines 10–12). The algorithm verifies the presence of new invariants at each step and adds them to the set C (line 13). The method terminates once the obtained place invariants have covered all places of the net (lines 14–15). The place invariant coverage implies that each place refers to a non-zero place - related entry of at least one invariant.

```
   Data: A Petri net N = (P, T, F, M⃗_0)
   Result: The net N is bounded or uncovered
 1 D ← an identity matrix of size |P|;
 2 A ← an incidence matrix of N;
 3 A^T ← Transpose Matrix (A);
 4 Q ← union matrix of D and A^T;
 5 C ← ∅;
 6 foreach column j of matrix A^T do
 7 |   find row pairs that their sum is equal to 0 and add it to Q;
 8 |   delete from Q rows that intersection with j-th column is not equal to 0;
 9 |   reduce redundant rows of Q;
10 |   foreach row i of matrix A^T do
11 |   |   if i = 0 then
12 |   |   |   I ← a place invariant that refers to the row i in D;
13 |   |   |   C ← C ∪ {I};
14 |   |   |   if C covers all places in the net N then
15 |   |   |   |   is_covered ← true;
16 |   |   |   |   break;
17 |   |   |   end
18 |   |   end
19 |   end
20 end
21 if is_covered then
22 |   print "net N is bounded";
23 else
24 |   print "net N is uncovered by place invariants";
25 end
```

Algorithm 4.2: The proposed boundedness analysis based on coverage

Example 4.2

Below in Fig. 2.2, boundedness with the proposed solution of the mulit-robot system specified by the Petri net N_1 is analyzed. According to Algorithm 4.2, initially the unit matrix $Q = [D|A^\mathsf{T}]$ is formed (line 4) as shown in Equation 4.2.

$$Q = \begin{array}{c} \begin{matrix} p_1 & p_2 & p_3 & p_4 & p_5 & p_6 & p_7 & p_8 & p_9 & t_1 & t_2 & t_3 & t_4 & t_5 & t_6 \end{matrix} \\ \left[\begin{array}{ccccccccc|cccccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & -1 & 0 & 0 \end{array}\right] \end{array} \tag{4.2}$$

In the next stage, the matrix Q is transformed (lines 7–9), while the subsequent transitions are examined (line 6). Rows of A^T are zeroed (marked in blue) (line 7). They will refer to the place invariants formed in the matrix D (marked in green). Calculated p-invariants are added to the set I (line 12). In the selected example, only three transitions out of six in total are required to be processed to obtain the place invariant coverage. Equation 4.3 describes the matrix Q after examination of the third transition i.e., behind three iterations of the method four p-invariants was computed. Clearly, invariants from the set I cover all places in the Petri net (lines 14–15). Note that every place $p \in \{p_1, \cdots, p_9\}$ has a corresponding non-zero entry in at least one p-invariant. The procedure terminates its execution (line 16) with the result that the Petri net N_1 is covered by place invariants (line 22), hence it is bounded.

$$Q = \begin{array}{c} \begin{matrix} p_1 & p_2 & p_3 & p_4 & p_5 & p_6 & p_7 & p_8 & p_9 & t_1 & t_2 & t_3 & t_4 & t_5 & t_6 \end{matrix} \\ \left[\begin{array}{ccccccccc|cccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{array}\right] \end{array} \tag{4.3}$$

Since a Petri net is covered by place invariants, the Petri net is bounded. We can notice that not all invariants are required to compute, but only as many as it is needed to cover the Petri net contrary to the typical method given in Algorithm 3.5. On the other hand,

in the worst case, the algorithm is obliged to compute all the invariants. And also, if the net is not covered, the boundedness is not determined. Therefore, the novel algorithm suggests finding its best application for covered Petri nets. Analyzing the boundedness of an uncovered Petri net in this way does not bring any changes. Hence, at early stage, it rises the need to quickly estimate whether a given Petri net could potentially be covered or uncovered by place invariants. This is the purpose of the next proposed method.

4.1.3 A Method Based on Reduced Row Echelon Form

The next proposed method uses common linear algebra matrix operations like Gauss-Jordan elimination [172] to solve linear systems. The search for invariants can be checked to solve the homogenous linear system $A\vec{x}$, where A is an incidence matrix of a Petri net. Section 2.7 states that one of possible methodologies to solve such a system is to adopt a matrix in the reduced row echelon form that can be obtained after Gauss-Jordan elimination. We also know that a Petri net covered by p-invariants is bounded (c.f. Chapter 3). Combining it, we shall propose a novel algorithm to support designers in checking the boundedness of Petri nets. It is particularly applicable in the preliminary determination of whether a Petri net cannot be covered by place invariants. The presented technique does not permit obtaining all-place invariants. Moreover, it operates in polynomial time.

Proposition 4.1. A Petri net cannot be covered by place invariants if there exists at least one row containing only a leading one in a matrix B, where the matrix B is in the reduced row echelon form of the incidence matrix A of the Petri net.

Proof. If there exists a row that contains only a leading one in matrix B, according to Def. 2.41 (solving linear systems $A\vec{x} = 0$ with application of matrix in the reduced row echelon form)

$$p_i = 0\,, \tag{4.4}$$

where i is the i-th column index of matrix B that correspond to the place i of a Petri net. Hence, place i is never covered since i-th entry of any $\vec{x}$ always equals to 0. Thus, the Petri net cannot be covered by p-invariants. □

Proposition 4.2. A Petri net cannot be covered by place invariants if there exists at least one row containing only ones in a matrix B, where the matrix B is in the reduced row echelon form of the incidence matrix A of the Petri net.

Proof. If there exists a row that contains only leading one and ones in matrix B, according to Def. 2.41 (solving linear systems $A\vec{x} = 0$ with application of matrix in reduced row echelon form)

$$p_i = -p_{j1} + (-p_{j2}) + \cdots + (-p_{jk})\,, \tag{4.5}$$

where i is the i-th column index with leading one and j-th k columns with ones. Hence, $p_i \leq 0$, place i is never covered since i-th entry of any $\vec{x}$ is always equal or less to 0. Thus, the Petri net cannot be covered by p-invariants. □

Important

It is worth noting that if a Petri net contains any transitions that do not have input places, the behavior of the methods is unexpected. A transition which has only output places generates tokens ad infinitum. Such nets are obliviously unbounded.

To disprove that a Petri net can be covered by place invariants, the proposed polynomial-time method is divided into following steps presented in Algorithm 4.3.

```
   Data: A Petri net N = (P, T, F, M⃗_0)
 1 A ← an incidence matrix of N;
 2 Function Perform Gauss-Jordan Elimination(A):
 3 |   B ← A;
 4 |   swap rows so that all rows with all zero entries are on the bottom;
 5 |   repeat
 6 |   |   swap rows so that the row with the largest, leftmost nonzero entry is on
   |   |    top;
 7 |   |   multiply the top row by a scalar so that top row's leading entry becomes 1;
 8 |   |   add/subtract multiples of the top row to the other rows so that all other
   |   |    entries in the column containing the top row's leading entry are all zero;
 9 |   |   swap rows so that the leading entry of each nonzero row is to the right of
   |   |    the leading entry of the row above it;
10 |   until all the leading entries are 1;
11 |   return B;
12 B ← Perform Gauss-Jordan Elimination(A);
13 foreach row i of the matrix B do
14 |   if i has only a leading one then
15 |   |   is_not_covered ← true;
16 |   |   break;
17 |   end
18 |   if i has only ones then
19 |   |   is_not_covered ← true;
20 |   |   break;
21 |   end
22 end
23 if is_not_covered then
24 |   print "net N is not covered by p-invariants";
25 else
26 |   print "net N might be covered by place invariants, therefore it may be
   |    bounded.";
27 end
```

Algorithm 4.3: A proposed invariants coverage disproving procedure

The idea of the proposed method is to perform Gauss-Jordan elimination in order to obtain the reduced row echelon form of the incidence matrix (lines 2–11). Then, we test

conditions that disprove that a Petri net can be covered by place invariants. Proposition 4.1 is tested on lines 7–10 of the algorithm and Proposition 4.2 is assessed on lines 11–14. If a Petri net is not p-invariant coverable, the proposed algorithm proves it in an efficient way i.e., polynomial time.

> Important

We know that a place invariant cover determines boundedness, however a Petri net that is not covered cannot be claimed to be unbounded.

On the other hand, we do not know whether a Petri net is covered (bounded) if none of the proposed condition appears. It is the main bottleneck of the novel solution. However, the experimental results show, and are subjected to detailed analyses in Chapter 5, that if a Petri net is not marked as uncovered by the algorithm, it is likely to be covered, i.e., bounded. Moreover, the information about the non-coverage of a Petri net by place invariants can be used to select methods for further analyses of boundedness. For not covered Petri nets, it is pointless to use solutions based on computation of invariants and then to use algorithms, for example, on reachability graphs instead.

Example 4.3

The next example shows a Petri net N_6 [1] in Fig. 4.1 that is bounded and not covered by p-invariants. The incidence matrix of the Petri net N_6 after Gauss-Jordan elimination in the reduced row echelon form is presented in Equation 4.6.

$$B = \begin{array}{c} \begin{matrix} p_1 & p_0 & p_2 & p_3 & p_4 & p_7 & p_5 & p_6 \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} \end{array} \tag{4.6}$$

The reduced row echelon form has three rows with just a leading one that $p_1 = 0, p_0 = 0, p_2 = 0$, thus places p_1, p_0, p_2 are not covered. Whereas, the reachability graph of the Petri net shown in Fig. 4.2 confirms that the net is bounded, where $k = 1$. Table 4.1 shows markings of every reachable state of the Petri net N_6.

An example of a place invariant uncovered but a bounded Petri net is described to indicate that an uncovered net does not imply unboundedness. However, it should be

[1] Available online: http://www.hippo.issi.uz.zgora.pl/index.php?id=petri_net&nr=292 (accessed on December 11, 2022)

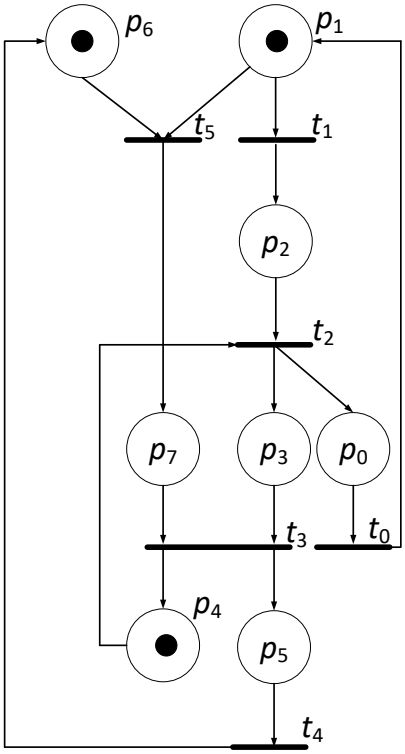


Figure 4.1: Non-p-invariant covered but the bounded Petri net N_6.

Table 4.1: Description of each reachable marking in N_6

	p_1	p_0	p_2	p_3	p_4	p_7	p_5	p_6
M_0	1	0	0	0	1	0	0	1
M_1	0	0	0	0	1	1	0	0
M_2	0	0	1	0	1	0	0	1
M_3	0	1	0	1	0	0	0	1
M_4	1	0	0	1	0	0	0	1
M_5	0	0	0	1	0	1	0	0
M_6	0	0	1	1	0	0	0	1
M_7	0	0	0	0	1	0	1	0
M_8	0	0	0	0	1	0	0	1

emphasized that the occurrence of such Petri nets is relatively rare. Even though experimental verification is conducted on hundreds various Petri nets in Chapter 5, the percentage of such cases is negligible.

Gauss-Jordan elimination constitutes a core of the proposed method. The run-time of the algorithm depends mainly on a chosen Gauss-Jordan implementation. Nowadays, extremely fast and efficient variants of Gaussian Elimination are available [50, 132, 134],

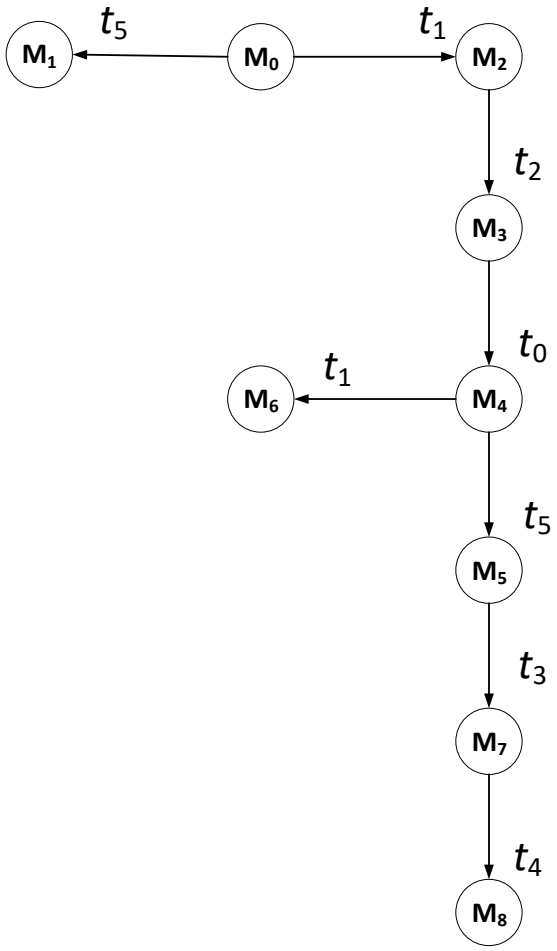


Figure 4.2: The state space of Petri net N_6.

for instance GPU-based [12]. Most mathematical tools (e.g., MATLAB [2]) offer quick numerical methods [3].

The computational complexity of the proposed algorithm is bounded by $O(|T|^2|P|)$, where $|T|$ is the number of transitions and $|P|$ is the number of places of a Petri net. The algorithm is divided into two parts. At first, transformation of the incidence matrix to the reduced row echelon form is executed at most $|T|^2|P|$ times. The second part with the outer loop and nested loops is executed at most $|T||P|$ times. The presented method shows that a Petri net that is non-covered by place invariants can be detected in polynomial time.

[2]MATLAB combines a desktop environment tuned for iterative analysis and design processes with a programming language that expresses matrix and array mathematics directly. More information available online: `https://www.mathworks.com/products/matlab.html` (accessed on December 11, 2022)

[3]For instance, *rref* function in MATLAB. More information available online: `https://www.mathworks.com/help/matlab/ref/rref.html` (accessed on December 11, 2022)

4.2 Safeness

In the proposed methods for checking safeness, we shall utilize two methodologies for computation of invariants with forming state machine components: exploration of reachability graphs or the linear algebra approach. Similar techniques, just like for verification of boundedness, are proposed for the analysis of safeness in order to improve time efficiency in comparison to the existing methods maintaining effectiveness (correctness of results). As explained in the previous chapter, state reachability, and the problem of searching for all minimal invariants, are time exponential in general. Hence, the proposed novel algorithms find their best application in particular situations. More details about the effectiveness and efficiency of the proposed methods will be introduced in Chapter 5. Experimental Verification of Proposed Methods.

4.2.1 A Method Based on Reachability Graphs

Safeness can be analyzed by means of exploration of reachability graphs. The construction of a full graph involves exponential time computational complexity, which we explained in detail in the previous chapter. It can be noticed that during the construction of the graph, the number of tokens for particular places is determined by subsequent markings. Hence, during the building of the state space, it is possible to check whether a Petri net can be safe, i.e., there is no such state in which even one place has more than one token. Therefore, the algorithm can be terminated without the need to designate all states the moment a place contains more than one token. Moreover, in the case of unbounded place detection, such a Petri net cannot also be safe, because if the condition in Equation 4.1 is satisfied then there exists a place that accumulates tokens indefinitely. Thus, the execution of the method may also be interrupted.

Below, a novel algorithm based on the construction of a reachability graph for analysis of safeness of concurrent control systems specified by Petri nets is presented. The core idea of this method during the construction of the graph is to verify whether there will be such a marking which contains more than one token in some place. In such a case, we can terminate the operation of the algorithm, claiming that the Petri net is unsafe.

Algorithm 4.4 presents a solution that implements the described idea. The proposed method may determine that a Petri net is not safe in the process of building state spaces. Of course, if a Petri net is safe, it is required to construct the entire reachability graph. Therefore, if a Petri net is suspected of being unbounded, e.g., by a method based on a reduced row echelon form, it may not be safe, since any unbounded net is also unsafe by definition. In that case, the use of this proposed algorithm may be the most advisable.

Initially, the proposed algorithm requests a Petri net N as an input. Then the initial marking $\overrightarrow{M_0}$ is assigned to the variable $\overrightarrow{M}$ (line 1). In the next step, a loop begins, which works until a new state cannot be designated (line 2). In the loop, after subsequent firing of the enabled transitions (line 3), a new marking is determined and assigned to the

```
   Data: A Petri net N = (P, T, F, M⃗_0)
   Result: The net N is safe or unsafe
 1 M⃗ ← an initial marking M⃗_0;
 2 while exists a new marking that can be reachable from M⃗ do
 3     foreach enabled transition t at M⃗ do
 4         M⃗_s ← a marking after t fired from M⃗;
 5         if M⃗_s ∉ 𝕄 then
 6             foreach place p in M⃗_s do
 7                 if M_s(p) > 1 then
 8                     is_safe ← false;
 9                     break;
10                 end
11             end
12             if exists M⃗_r on the path from M⃗_0 that ∀p ∈ P, M_s(p) ≥ M_r(p) then
13                 if ∀p ∈ P, M_s(p) > M_r(p) then
14                     M_s(p) ← ω;
15                     is_safe ← false;
16                     break;
17                 end
18             end
19             𝕄 ← 𝕄 ∪ {M⃗_s};
20             Σ ← Σ ∪ {t};
21         end
22     end
23 end
24 if is_safe then
25     print "net N is safe";
26 else
27     print "net N is unsafe";
28 end
```

Algorithm 4.4: The proposed safeness analysis on reachability graph

variable $\overrightarrow{Ms}$ (line 4). If there is a place in this marking where there is more than one token (line 7) then the loop is broken (line 9) and the algorithm terminates with the result the Petri net N being unsafe (line 27). Similarly, the algorithm terminates if an ω is assigned to a place, i.e., a place that will accumulate tokens indefinitely (line 14). Marking a place as unbounded is based on a condition from the Equation 4.1 (see Section 4.1 for more details). Otherwise, if no unsafe place occurs, the algorithm continues by firing the next enabled transition (line 3) and designating a new place until there is no new unique marking (line 23). The Petri net N is safe (line 25) when each place in any reachable marking has at most one token.

4.2.2 A Method Based on Invariants

Another method introduces an algorithm for analyzing safeness based on the computing of place invariants that form correct state machine components. It is claimed (q.v. Section 3.4) that a Petri net covered by SM-components is safe. As already mentioned in Section 3.2, a Petri net covered by place invariants is bounded, and if it is also covered by SMCs, it is safe. In view of the above, the proposed algorithm for the analysis of boundedness was extended to check whether the obtained invariant composes the correct SMCs. That is, whether a SM-net has exactly one token in the initial marking. When all the state machine components cover the Petri net, i.e., every place of the Petri net is a part of at least one SMC, then the algorithm stops its operation with the result of the Petri net being safe. The method is first described, after which it is explained by its profits and limitations. The novel solution can be presented by the steps shown in Algorithm 4.5.

```
   Data: A Petri net N = (P, T, F, M⃗_0)
 1 A ← an incidence matrix of N;
 2 D ← an identity matrix of size |P|;
 3 A^T ← Transpose Matrix (A);
 4 C ← ∅;
 5 Q ← union matrix of D and A^T;
 6 foreach column j of matrix A^T do
 7 |   find row pairs that their sum is equal to 0 and add it to Q;
 8 |   delete from Q rows that intersection with j-th column is not equal to 0;
 9 |   reduce redundant rows of Q;
10 |   foreach row i of matrix A^T do
11 |   |   if i = 0 then
12 |   |   |   I ← a place invariant that refers to the row i in D;
13 |   |   |   if I forms a proper SMC (contains exactly one token in M⃗_0) then
14 |   |   |   |   C ← C ∪ {I};
15 |   |   |   end
16 |   |   end
17 |   |   if C covers all places then
18 |   |   |   is_SMCs_covered ← true;
19 |   |   |   break;
20 |   |   end
21 |   end
22 end
23 if is_SMCs_covered then
24 |   print "net N is safe";
25 else
26 |   print "net N is uncovered by SMCs";
27 end
```

Algorithm 4.5: The proposed SMCs cover-based boundedness analysis

Firstly, initialization is performed to read an incidence matrix of a Petri net N. In addition, the necessary variables: D — an identity matrix, A^{T} — the transposed incidence

matrix, C — empty set for SMC cover are set (lines 1–4). At the next stage, the unit matrix Q is formed as a conjunction of the identity matrix D, and the transposed incidence matrix of the Petri net system A^{T} (line 5). Note that D will hold the place invariants during examination of subsequent transitions (lines 7–9). If a row in A^{T} is zeroed (all entries are equal to zero) then the correct p-invariant can be read from the corresponded row in matrix D. Checking for the state machine cover is a key objective of the algorithm. This step is executed for each transition until the SM-cover is found. At each stage, the algorithm verifies whether the obtained place invariants I form a proper state machine component (lines 12–13). A correct S-component contains exactly one token in the initially marked place. Then, SM-component is added to set C (line 14). Simultaneously, the existence of the SMC cover, understood as state machine components which cover all places of the net, is analyzed. The execution of the method is terminated when the coverage is gained (lines 17–20) with the result that the net is safe (lines 23–24).

Otherwise, all state machine components are obtained, and the algorithm informs about the absence of coverage. If a Petri net is not covered by S-components, safeness is not determined by this algorithm, because a not covered net may be both safe or unsafe. Moreover, the result may not be obtained in reasonable time as all p-invariants are required to compute.

Below, the proposed method is presented on the example of the Petri net N_1, which models the operation of the multi-robot shown in Fig. 2.2. The first step of the algorithm is to input the Petri net as the incident matrix A_1 shown in Equation 2.16. Then the incidence matrix is transposed and united with the identity matrix D. That matrix is presented in Equation 4.2. Successively, for each column j in the matrix A^{T}, such pairs of rows are sought that their sum is equal to zero. Those rows are added to the Q matrix, while redundant rows and those whose intersection with j column is not equal to zero are removed from the matrix Q. In the next step, we check whether any row in the matrix A^{T} has been zeroed, if so, the corresponding row in the matrix D forms a place invariant. After examination of three transitions (three iterations: $j = \{1, 2, 3\}$), the matrix Q is given in Equation 4.3. In the presented example there are four such place invariants in the set I:

$$\begin{aligned} I(1) &= [0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1] \\ I(2) &= [1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0] \\ I(3) &= [0\ 0\ 1\ 0\ 0\ 1\ 1\ 0\ 0] \\ I(4) &= [0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0]\,. \end{aligned} \tag{4.7}$$

Simultaneously, we verify whether the obtained invariant composes the correct state machine component, that is, whether it has exactly one token in the place marked initially (marked in blue), if so, SMC is added to set C. After each obtained S-component, the SMC-cover is checked. The coverage is established after three out of six transitions have been examined. The loop is broken, and the algorithm ends with information that the Petri net N_1 is safe.

4.3 Conclusions

The proposed novel algorithms are meant to bridge the existing gap. The described proposals for the analysis of boundedness and safeness are based on two approaches: state space exploration or linear algebra (invariants computation). Three new methods of verification of boundedness and two of safeness have been proposed. The problems of constructing reachability graphs and calculating all minimum invariants are exponential with respect to time. Hence, the novel proposed methods bring improvements in special cases. A detailed analysis of the efficiency and effectiveness of the solution is presented in Chapter 5. It is worth adding that the method which applies an incidence matrix in the reduced row echelon form matrix can reject place invariants coverage in polynomial time or determine with high probability that it is covered, i.e. potentially bounded. The presented solutions terminate the operation of the algorithms regardless of the conditions confirming or rejecting Petri net properties being met. Some bottlenecks of the novel methods are also described in the next chapter, two main limitations being: a Petri net that is not covered by place invariants is not always unbounded and a net which is uncovered by S-components can also be safe.

Chapter 5

Experimental Verification of the Proposed Methods

Experimental research is an essential element in developing novel algorithms as well as in testing already available solutions for their optimal application. Each proposed algorithm should be evaluated by means of reference methods in terms of its correctness and operating time. The algorithms introduced in Chapter 4 were verified experimentally. The conducted research was meant to compare the presented methods to reference algorithms. Here, we also discuss the best application of the selected algorithms in order to verify boundedness and safeness. The benchmark library used in the experiments, containing 234 Petri nets, is part of a Hippo project developed at the University of Zielona Góra. The set of test modules consists of various classes as well as features various levels of complexity of the Petri nets used for the specification of hypothetical and real-life examples of control systems.

Our research focuses on the inspection of the effectiveness and efficiency of the selected algorithms. As already stated in previous chapters, effectiveness is understood here as consistency of results, and efficiency as time needed to achieve a result. In the case of methods based on the calculation of invariants, the amount of system memory usage is not an obstacle at this stage. Exponentiality with respect to memory is an important optimization problem within state space construction. However, as we must first deal with time complexity, memory performance tests are not part of this work. A comparison of algorithms allowed us to indicate the advantages and disadvantages of the presented methods as well as to indicate the best cases of their use.

Below, the Hippo system which was used in the experimental research is introduced. The introduction is followed by comparison of the proposed analyzes of boundedness algorithms to reference methods. Later, safeness methods are scrutinized and finally, the summary of methods that are optimal to apply in particular cases is presented.

5.1 The Hippo System

There are various available tools on the market to work with Petri nets. Some of this software is probably known only to its authors, other is problematic even in its installation or launching. Only some is suitable for daily use by end users, i.e. designers of concurrent control systems based on Petri nets. In response to this demand, Hippo [1] [157] was launched in 2005, designed as a set of stand-alone tools for broad Petri net analysis. Initially, the components were oriented onto systems based on the hypergraph theory. The computer-aided design system has been significantly developed for years. Now, it focuses on control systems specified by a Petri net. The most recent version of Hippo consists of a set of programs that support designers of control systems, including prototyping, verification, analysis, decomposition, modeling, and automatic code generation (implementation). Additionally, since they provide balance between optimal results and computational time, the modules allow solving the same problems by various methods.

Hippo tools include three completely different approaches to the analysis (or decomposition) of control systems. They can be used depending on one's needs. In the previous chapters, we stated that there was not one perfect analytical method in general case of Petri nets. Therefore, the following techniques are available:

- linear algebra approach,
- algorithms based on graph theory,
- algorithms based on hypergraph theory.

Below, methods included in Hippo are presented. Moreover, the Hippo project consists of the library of currently 243 various Petri nets. The software accepts input as a PNH [2] file. The format includes an incidence matrix of a Petri net and an initial marking vector. Additionally, other options can be specified. However, they are optional for analytical proposes. Hippo permits verification by classification of Petri nets, analysis of concurrency and sequentiality relations. Furthermore, computation of place invariants and safeness and boundedness checks of Petri nets by various algorithms are implemented. Finally, the Hippo system can return the model in one of the following formats: Verilog (for the implementation in FPGA), PNG (the graphical representation of the model) and XML (for PIPE).

[1] The detailed description of the project and access to Petri net test modules can be found online: `http://www.hippo.issi.uz.zgora.pl` (accessed on December 11, 2022)

[2] Petri Net Hippo format

5.2 Boundedness

The novel algorithms for boundedness analysis have been introduced in Section 4.1. Three experiments were performed. The first one compares methods based on reachability graphs. The second report exposes algorithms based on linear algebra, i.e., computation of invariants of places. Sample results of the conducted research are shown in Table 5.1 (reachability graph-based algorithms) and in Table 5.2 (invariants coverage-based algorithms). Furthermore, the third experiment, i.e. a method to estimate which of the proposed algorithms should be used in the particular case, is shown in Table 5.3. The full results of all experiments are attached in Appendix A. The tests were prepared by means of a dedicated computational server equipped with an Intel(R) Xeon(R) Gold 5220 @2.2 GHz processor and 128 GB of RAM.

First of all, a fundamental difference between methods based on browsing state space and computation of place invariants can be identified. An overview of all possible states returns the answer whether a Petri net is bounded or unbounded. On the other hand, algorithms based on computing invariants are usually faster (shorter run-time). In the case of analysis of invariants, we talk about their coverage of a Petri net. The coverage guarantees the boundedness of a Petri net; however, lack of coverage does not indicate that the net cannot be bounded. Generally, such cases are rare. In our test library, there are fourteen such Petri nets out of 243, which is less than 6%. Of the selected Petri nets shown in the tables, *balduzzi1* is an example an uncovered and bounded Petri net.

Table 5.1 compiles reachability graph-based algorithms for boundedness verification. The full-graph construction reference method presented in Algorithm 3.1 and its implementation for the purpose of the experimental research is the first reference method (q.v. Algorithm 3.4). It is used to compare the first proposed algorithm. The first proposal with the reduced state space (terminating if an unbounded place is located) is given in Algorithm 4.1. Both of them are based on browsing the space of possible reachable states. The marking *n/a* indicates that a result was not obtained within assumed time i.e. one hour. The particular columns contain the following values:

- Name — the name of a Petri net (benchmark),
- $|P|$ — the number of places in the Petri net,
- $|T|$ — the number of transitions in the Petri net,
- bounded — whether the Petri net is bounded,
- runtime — the run-time of the respective methods in milliseconds,
- 1st reference method — a full reachability graph-based reference algorithm,
- 1st proposed method — a proposed reduced reachability graph-based algorithm.

Table 5.1: Sample results of experiments, state space-based algorithms

Name	$\|P\|$	$\|T\|$	1st reference method		1st proposed method	
			bounded	runtime[ms]	bounded	runtime[ms]
coloured	4	3	true	25.564	true	17.262
cortadella1	9	6	true	29.867	true	17.477
board_game	13	13	false	21.418	false	15.285
lnet_p6n2	14	16	false	1,316.918	false	49.137
dining_philosophers	15	10	false	124.836	false	45.724
brenner1	16	12	true	51,507.596	true	49,074.995
state_space16	16	16	true	3,208.890	true	1,802.107
automation3	17	16	true	66,731	true	78.435
balduzzi1	18	16	true	31.694	true	30.205
adam1	24	12	true	57.150	true	55.183
HAN	24	40	true	100,185.767	true	50,454.547
crossroadSM_FPGA	32	12	true	79.008	true	30.463
state_space32	32	32	n/a	n/a	n/a	n/a
state_space48	48	48	n/a	n/a	n/a	n/a
cn_crr7	56	15	true	191,885.115	true	95,667.592
cn_crr10	80	21	n/a	n/a	n/a	n/a
cn_crr15	120	31	n/a	n/a	n/a	n/a
cn_crr25	200	51	n/a	n/a	n/a	n/a

Computational complexity, which is not directly dependent on the number of places and transitions in a Petri net [89], is what should be considered when comparing the algorithms of boundedness property analysis based on state space. The time needed to verify boundedness of a Petri net depends on the number of reachable states. This is illustrated in Table 5.1 by an example of a Petri net with fourteen places and sixteen transitions *lnet_p6n2*. The first reference method takes more than one second (1,316.918 ms) to obtain the result, whereas for the larger Petri net *crossroadSM_FPGA*, it is just seventy-nine milliseconds.

The first proposed method is applicable to Petri nets that are unbounded since the result can be obtained quickly by interrupting the operation of the algorithm after detecting an unbounded place. This can be seen in the example of the Petri net *lnet_p6n2*, for which it took 1.3 seconds (1,316.918 ms) to build the entire reachability graph and view all the states (the 1st reference method), while the determining of the state space and simultaneous verification of the places for unboundedness was performed almost twenty-seven times faster (the 1st proposed method). Since the Petri net is not bounded and unbounded, the place was detected early. For bounded Petri nets, the result time is close to the reference method. Although with larger Petri nets, we can see some improvement resulting from the fact that the checking of boundedness of places occurs already during the construction of the reachability graph. Since there is no need to browse the designated state space, the absence of unbounded places means that the Petri net is bounded. The *HAN* Petri net, in which the total time of the property analysis is two times shorter

compared to the reference method, is a good example of this. Hence, a solution which can estimate whether a Petri net may not be bounded at the beginning of the boundedness analysis procedure is needed. The third proposed method is dedicated to such cases. In the case of potentially unbounded Petri nets, a beneficial solution seems to choose the first proposed method. On the other hand, it is more efficient to use the other method (the next proposed method is discussed in the following paragraph) for bounded nets. Generally, state space analysis makes it possible to answer whether a Petri net is bounded or unbounded. However, with current methods, computational complexity of the reachability graph construction makes it impossible to complete it for some even relatively small, bounded Petri nets (several places and transitions) as: *state_space32*, *state_space48*, *cn_crr10*, *cn_crr15* or *cn_crr25*.

Table 5.2 presents research on algorithms that analyze the property of boundedness based on the verification of Petri net coverage by invariants of places. The Martinez-Silva (M-S) algorithm (q.v. Algorithm 3.3) is a classic popular method for computation of all minimal invariants (the full set), and it is presented at length in Section 3.2. Here, it is an implemented analysis of boundedness of the Petri net (q.v. Algorithm 3.5). The examined method is the second proposed algorithm shown in Algorithm 4.2. The particular columns describe the following values:

- covered — whether the Petri net is covered by place invariants therefore bounded,
- runtime — the run-time of the respective methods in milliseconds,
- 2nd reference method — the reference classic (full invariants set) algorithm implemented toward p-invariants coverage verification,
- 2nd proposed method — the proposed algorithm based on terminating operation while the place invariants coverage is obtained (set of reduced invariants).

Table 5.2 compares the two methods based on invariants computing differently than by generating state space. The compared algorithms check the coverage of a Petri net with the invariants of places. The second reference method is a classic algorithm for obtaining all minimal invariants in a Petri net. In our research, it was adapted to the Petri net coverage for the requested boundedness analysis. The guarantee of correctness of the obtained result is an undoubted advantage. However, even with relatively small Petri nets, such as *crossroadSM_FPGA* (32 places, 12 transitions), the time needed to perform the analysis is almost 14 minutes (891,400.791 ms). And for *cn_crr7* and larger nets, the run-time is more than one hour, so after that time the algorithm was terminated (not found the result in the assumed time). In such a case, we claim that the method is inefficient.

The second proposed method, while calculating the invariants, checks whether a Petri net has already been covered by them. On the one hand, this is associated with a greater number of conditions. Thus, for exceedingly small nets, for example *colored* or *cortadella1*,

Table 5.2: Sample results of experiments, linear algebra-based algorithms

Name	$\|P\|$	$\|T\|$	2nd reference method		2nd proposed method	
			covered	runtime[ms]	covered	runtime[ms]
coloured	4	3	true	0.045	true	0.088
cortadella1	9	6	true	0.065	true	0.124
board_game	13	13	false	0.104	false	0.237
lnet_p6n2	14	16	false	0.085	false	0.240
dining_philosophers	15	10	false	0.068	false	0.331
brenner1	16	12	true	0.095	true	0.519
state_space16	16	16	true	0.091	true	2.167
automation3	17	16	false	0.083	false	0.567
balduzzi1	18	16	false	0.107	false	0.656
adam1	24	12	true	1.805	true	1.884
HAN	24	40	true	0.102	true	1.235
crossroadSM_FPGA	32	12	true	891,400.791	true	1.449
state_space32	32	32	true	0.264	true	2.254
state_space48	48	48	true	0.375	true	8.344
cn_crr7	56	15	n/a	n/a	true	3.832
cn_crr10	80	21	n/a	n/a	true	12.782
cn_crr15	120	31	n/a	n/a	true	37.731
cn_crr25	200	51	n/a	n/a	true	206.393

the time is slightly higher for this algorithm. On the other hand, the difference is essential for larger nets, e.g. *crossroadSM_FPGA* as much as 4,474 times faster. For *cn_crr7* and nets with more places and transitions, there is a result, as opposed to no result with the reference method. However, the operating time on the *cn_crr25* net with two hundred places and fifty-one transitions is more than one minute. The computational complexity of this algorithm is obviously exponential, i.e. in the worst case, it requires to obtain all the minimal invariants, especially when a Petri net is not covered. In such a case, the next proposed method may deliver the result in an instant.

Table 5.3 presents results of tests of the third proposed algorithm presented in Algorithm 4.3. To our best knowledge, it is currently the fastest method of an approximate examination of Petri net coverage by place invariants. Although the algorithm, as proven, works in polynomial time, it only guarantees that a Petri net is uncovered by place invariants. Otherwise, it is highly likely that a Petri net may be covered by place invariants, but this has not been confirmed. Nevertheless, we have not found such a case in our extensive experimental research, which does not mean that it does not exist. When informed that a Petri net is not covered, the designer receives information that it is highly probable that the net is not correctly designed. Even if it can be bounded, it is not live. In the reverse case, the designer cannot be sure of the boundedness of the Petri net. However, this may be the only solution capable of any property analysis for larger nets, i.e. such with more than hundreds of places.

Table 5.3: Sample results of experiments, the reduced row echelon form-based algorithm

Name	$\|P\|$	$\|T\|$	3rd proposed method		runtime[ms]
			covered	further analysis	
coloured	4	3	possibly	2nd proposed method	0.003
cortadella1	9	6	possibly	2nd proposed method	0.003
board_game	13	13	false	1st proposed method	0.007
lnet_p6n2	14	16	false	1st proposed method	0.008
dining_philosophers	15	10	false	1st proposed method	0.006
brenner1	16	12	possibly	2nd proposed method	0.010
state_space16	16	16	possibly	2nd proposed method	0.008
automation3	17	16	false	1st proposed method	0.010
balduzzi1	18	16	false	1st proposed method	0.009
adam1	24	12	possibly	2nd proposed method	0.008
HAN	24	40	possibly	2nd proposed method	0.006
crossroadSM_FPGA	32	12	possibly	2nd proposed method	0.012
state_space32	32	32	possibly	2nd proposed method	0.035
state_space48	48	48	possibly	2nd proposed method	0.100
cn_crr7	56	15	possibly	2nd proposed method	0.026
cn_crr10	80	21	possibly	2nd proposed method	0.064
cn_crr15	120	31	possibly	2nd proposed method	0.193
cn_crr25	200	51	possibly	2nd proposed method	0.832

The third novel algorithm uses Gauss-Jordan elimination and is based on a matrix in a reduced row echelon form. It is mainly used to determinate which method for further boundedness analysis to choose: the exact, i.e. the first proposal based on a reduced reachability graph or the approximate one, i.e. the second proposal on coverage of invariants (the reduced set). The algorithm indicates whether a Petri net is not covered by place invariants, hence it is probably unbounded and then the first proposed method may effectively and efficiently check it. In addition, the rows with only ones in the reduced row echelon matrix obtained as a mid-result of this algorithm indicate potential unbounded places. Otherwise, it is highly likely that the Petri net is covered, and in that case the second proposed algorithm can be used to accomplish the coverage (confirming the boundedness).

The final findings of the extensive research on the boundedness property are presented in Table 5.4 as a toolchain combining the three novel methods examined above. The third proposed algorithm operating in polynomial time performs a preliminary boundedness analysis of a Petri net. If the Petri net is uncovered, then the use of methods based on invariants cover terminates with information about unresolved boundedness. Then we should apply the first proposed method based on searching the reachability graph. In the case of uncovered Petri nets, it is probable that the net has unbounded places, hence it will be unnecessary to build the entire graph. In the case of a potentially covered Petri net, it is optimal to use the second novel algorithm based on invariants cover. As the results show, it was achievable to investigate the boundedness property for all 243

Petri nets. Therefore, we confirm our thesis that it is possible to examine boundedness efficiently and effectively (within the rules we have set).

Table 5.4: Sample results of experiments, combined proposed methods

Name	$\|P\|$	$\|T\|$	bounded	Toolchain applied methods	runtime[ms]
coloured	4	3	true	3rd + 2nd prop. method	0.091
cortadella1	9	6	true	3rd + 2nd prop. method	0.127
board_game	13	13	false	3rd + 1st prop. method	15.292
lnet_p6n2	14	16	false	3rd + 1st prop. method	49.145
dining_philosophers	15	10	false	3rd + 1st prop. method	45.730
brenner1	16	12	true	3rd + 2nd prop. method	0.529
state_space16	16	16	true	3rd + 2nd prop. method	2.175
automation3	17	16	true	3rd + 1st prop. method	78.445
balduzzi1	18	16	true	3rd + 1st prop. method	30.214
adam1	24	12	true	3rd + 2nd prop. method	1.892
HAN	24	40	true	3rd + 2nd prop. method	1.241
crossroadSM_FPGA	32	12	true	3rd + 2nd prop. method	1.461
state_space32	32	32	true	3rd + 2nd prop. method	2.289
state_space48	48	48	true	3rd + 2nd prop. method	8.444
cn_crr7	56	15	true	3rd + 2nd prop. method	3.858
cn_crr10	80	21	true	3rd + 2nd prop. method	12.782
cn_crr15	120	31	true	3rd + 2nd prop. method	37.924
cn_crr25	200	51	true	3rd + 2nd prop. method	207.225

5.3 Safeness

The two algorithms for safeness analysis proposed in Section 4.2 were verified experimentally. The effectiveness (the proper result) and efficiency (the method run-time) of the proposed algorithms were evaluated as a key objective of the research. Novel methods were compared to their reference method, based on exploration of the reachability graph and computation of invariants, respectively. The fourth proposed algorithm (q.v. Algorithm 4.4) explores a reachability graph for an unsafe place. When such a place is found, the algorithm terminates with a result of a Petri net being unsafe. The Martinez-Silva (M-S) algorithm for calculation of invariants presented in Algorithm 3.3 was encouraged to obtain state machine cover as the fifth reference method (q.v. Algorithm 3.7). The fifth proposed method described in Algorithm 4.5 is based on computation of place invariants and focused on efficiency. It searches for invariants that form correct state machine components and after each obtained subsequent component, it verifies whether a Petri net has already been covered by those SMCs. If the Petri net is covered by state machine components, the method terminates, and it is claimed that the net is safe.

The algorithms were evaluated on the same dedicated computational server as for boundedness experiments. Table 5.5 (reachability graph-based) and Table 5.6 (invariants-based) show the results of experiments for selected benchmarks. The full report of the experiments is attached in Appendix A. The particular columns of tables contain the following values:

- Name — the name of a Petri net,
- $|P|$ — the number of places in a Petri net,
- $|T|$ — the number of transitions in a Petri net,
- safe — whether the Petri net is safe,
- runtime — the run-time of the respective methods in milliseconds,
- 4th reference method — the reachability graph-based reference algorithm,
- 5th reference method — the reference classic M-S algorithm for state machine components cover verification,
- 4th proposed method — the proposed algorithm based on termination operation when an unsafe place is found,
- 5th proposed method — the proposed algorithm based on terminating operation when SMCs coverage is obtained.

Table 5.5: Sample results of safeness experiments, reachability graph-based algorithms

Name	$\|P\|$	$\|T\|$	4th reference method		4th proposed method	
			safe	runtime[ms]	safe	runtime[ms]
kovalyov92	4	6	true	20.126	true	15.358
net1	6	5	true	29.288	true	16.399
CNC_machine	7	5	true	17.480	true	17.681
miczulski1	9	8	true	24.447	true	19.300
franczok1	10	14	true	32.773	true	18.367
Elevator01	12	17	false	75.668	false	64.379
board_game	13	13	false	21.769	false	14.392
hee1	14	12	true	40.923	true	56.927
silva5	16	8	true	52.875	true	36.178
hulgaard1	19	12	true	41.042	true	27.289
ConsistentExample	29	25	false	91.635	false	45.275
zuberek1	30	22	true	281.995	true	102.722
s_net_copy_mill	32	29	false	41,407.140	false	145.046
zuberek5	43	34	true	557.506	true	146.174
cn_crr15	120	31	n/a	n/a	true	n/a

The compared algorithms are divided into two groups. The first one focuses on methods based on reachability graphs. They are dedicated to the criterion of effectiveness, as the methods obtain an exact result, i.e. a Petri net is safe or unsafe. The fourth proposed method makes it possible to shorten the time needed to gain the result by simultaneously building a graph and searching for an unsafe place. Table 5.5 presents experimental research comparing the fourth reference method to the fourth proposed method. For example, in the case of the unsafe Petri net *s_net_copy_mill*, a classic solution generates a result after 41 seconds (41,407.140 ms) whereas the proposed method returns a result in less than 0.2 s (145.046 ms). This novel algorithm finds a key application in the initial phases of designing systems described by Petri nets. It often happens that a designer makes a mistake in the first versions of the specification (e.g. incorrect synchronization to places – one of the most popular mistakes made by designers) and then the designed Petri net is not safe. The proposed method makes it possible to show in an efficient way that a Petri net is unsafe. It is worth noting that also in the case of a safe Petri net, the operating time of the proposal is slightly shorter, especially in the case of larger nets (e.g. *hulgaard1*, *zuberek5*). This is because the reachability graph is explored during its construction, so there is no need to examine the entire graph after its construction performed by means of typical available methods including the reference method (construct the reachability graph and then read the properties of the Petri net).

Table 5.6: Sample results of safeness experiments, linear algebra-based algorithms

Name	$\|P\|$	$\|T\|$	5th reference method		5th proposed method	
			covered	runtime[ms]	covered	runtime[ms]
kovalyov92	4	6	true	0.026	true	0.096
net1	6	5	true	0.120	true	0.123
CNC_machine	7	5	true	0.035	true	0.086
miczulski1	9	8	true	0.058	true	0.138
franczok1	10	14	true	0.157	true	0.203
Elevator01	12	17	false	0.090	false	0.386
board_game	13	13	false	0.107	false	0.345
hee1	14	12	true	0.489	true	0.609
silva5	16	8	true	1.449	true	0.667
hulgaard1	19	12	true	11.882	true	1.194
ConsistentExample	29	25	false	56,220.855	false	964.609
zuberek1	30	22	true	12.602	true	3.973
s_net_copy_mill	32	29	false	0.444	false	0.642
zuberek5	43	34	n/a	n/a	true	2.726
cn_crr15	120	31	n/a	n/a	true	45.246

The second group is made up of algorithms based on linear algebra. Those methods are geared towards the criterion of efficiency. The result is not an exact: safe or unsafe, but safe or uncovered by state machine components. It is known from previous explanations and statements that a Petri net is safe if it is covered by SMCs. Otherwise, solutions

based on these approaches is inconclusive. Nevertheless, it should be added that there are relatively few cases of safe and uncovered Petri nets. In our test library of 243 Petri nets, there are twelve such cases, so they account for less than 5% of all benchmarks.

The fifth reference method implementing the classic Martinez-Silva algorithm is compared to the fifth proposed algorithm. The proposal computes a reduced set of place invariants and forms state machine components that cover a Petri net. Hence, it is not required to obtain all the (minimum) invariants that compose the correct SMCs, but only those necessary for coverage. As experimental research shows in Table 5.6, the results obtained by the fifth proposed algorithm are lower in time compared to the reference method. For small Petri nets, e.g., *kovalyov92*, *net1*, *CNC_machine*, the difference can be insignificant or even negative, as additional coverage checking after each state machine component can increase the running time. However, especially for larger Petri nets, such as *ConsistentExample*, the difference is significant. Moreover, for such large Petri nets as *zuberek5* or *cn_crr15* the algorithm is still efficient in contrast to the reference method that does not obtain a result in the assumed time, i.e. one hour.

To conclude the experimental results on the property of safeness, the fourth proposed method based on the reachability graph is dedicated to effective rejection of Petri net safeness, while the fifth proposed algorithm makes it possible to efficiently analyze large (several or more place and transitions) Petri nets covered by state machine components. The limitation of the fourth proposal is that due to the requirement of building the entire graph in the case of a safe Petri net, it may not be efficient for large size nets. On the other hand, the novel method using the linear algebra technique does not present a significant gain with uncovered Petri nets, since it will be necessary to obtain all SMCs anyway. Hence, a problem arises as to which method to use in a particular case. The third proposed method from the analysis of boundedness can be helpful here to estimate in polynomial time whether a selected Petri net can be covered by place invariants, as by definition, an unbounded Petri net cannot be safe. If a Petri net is uncovered by p-invariants, it is probably unbounded, hence also safe. Thus, the fourth proposed method may find its best application. Otherwise, it is highly likely that the Petri net is covered and therefore bounded. In this case, the fifth proposed method is effective, as it confirms whether a Petri net is not only bounded, but safe at the same time.

Chapter 6

Case Study of Petri Net-Based Specification of Concurrent Control Systems

This chapter deals with a case study of a Petri net-based specification of a manufacturing system. A Petri net used for the specification is required to be safe thus also bounded. Additionally, the net features input and output ports to communicate with the environment. Strictly speaking, it is an interpreted net. The aim of this chapter is to introduce one of the manufacturing systems and to illustrate the proposed analytical methods on a real-life example. The manufacturing system was proposed in [163]. The analyzed system may be found in a production company that belongs to a group of small and medium-sized enterprises of western Poland. They specialize in manufacturing and providing services to other industrial companies. The business operates in the field of processing large-format materials by means of laser and plotter technology.

6.1 Specification

The flat bar manufacturing process consists of nine points as described in [163].

1. Initially, a customer reports a demand for a product with specified parameters.

2. Documentation provided by the customer is studied. Designers assess the feasibility of the order. At this stage, the documentation is accepted, or changes are made after consultation with the customer.

3. The approved documentation is delivered to the implementation team, material is ordered in order to manufacture a flat bar of a length specified by the customer within the range from 3 to 6 meters.

4. The material received from the warehouse which fails to meet the requirements is adapted to specifications. The processed material is transferred to a band saw station.

5. The operator cuts the material accordingly. Production waste is stored in the post-production waste storage.

6. The semi-finished product is transported for milling. Material scraps formed during the milling process are stored in a special place.

7. The quality of the resulting product is controlled. Produce that does not meet the requirements is disposed as waste. Produce that requires corrections is subjected to additional processing. Produce compliant with quality parameters is transported to the pallet packing station.

8. Products are packed on pallets, as well as other production activities are conducted.

9. Finally, the ready pallets are handed over to the customer.

With reference to the informal specification, a model of the described manufacturing system on a Petri net was prepared. The Petri net-based specification of the system is shown in Fig. 6.1. This Petri net consists of twenty-four places and eighteen transitions along with four input signals and twenty output signals. Input signals active under some conditions are assigned to the transitions and described in Table 6.2. On the other hand, output signals that activate certain actions are related to places and presented in Table 6.1. A transition is enabled as defined by Def. 2.20, including the fulfillment of logical input conditions assigned to it. And the output signals connected to a given place are activated when the token is in that place. Note that for this reason the net must be safe.

6.2 Boundedness and Safeness Analysis

Below, a Petri net specifying the introduced manufacturing system by means of proposed algorithms is analyzed. It is worth noting at the outset that the analysis of boundedness and safeness by means of popular CAD tools, such as PIPE and IOPT-Tools, resulted in failure due to problems described in this dissertation. Let us begin with the analysis of boundedness, which will be followed by the analysis of safeness.

At the beginning, we used the 3rd proposed method (q.v. Algorithm 4.3) which revealed which algorithm to choose for optimal verification. This algorithm informs in polynomial time that a Petri net is uncovered by place invariants, so there are probably unbounded places in it. Hence, the 1st proposed method (q.v. Algorithm 4.1) on the reachability graph is selected. This algorithm interrupts the construction of the graph at the first unbounded place found, i.e., p_{12}. This way, we know that the Petri net is unbounded, and therefore also unsafe. Places p_{12}, and p_{16} (marked in red in Fig. 6.1) do not have output transitions. This is one of frequent mistakes made by designers. The

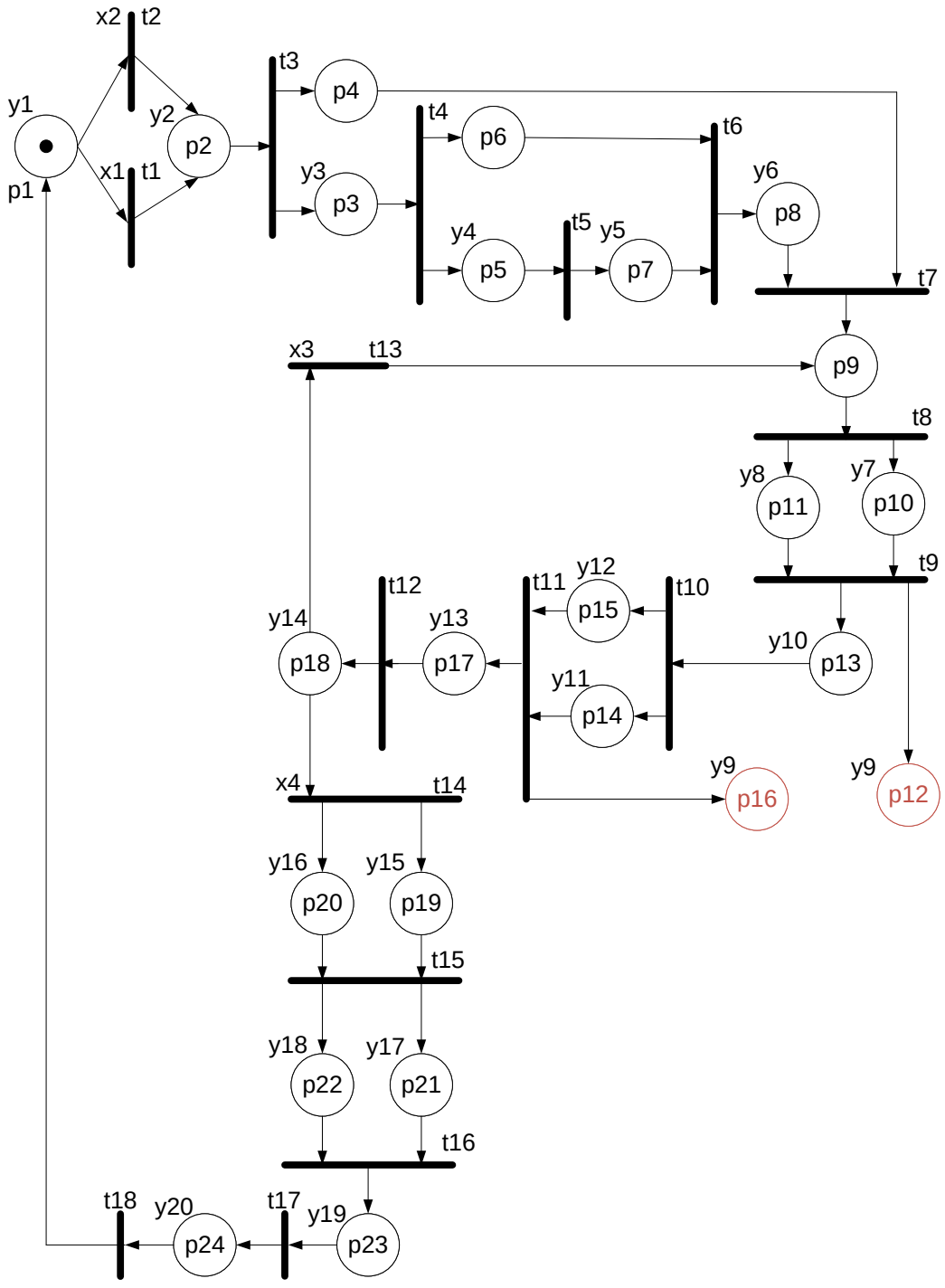


Figure 6.1: The initial specification of a real-life manufacturing system

Table 6.1: Description of output signals

Output Signal	Assigned to	Task
y_1	p_1	Request a product.
y_2	p_2	Submit the material order to the warehouse.
y_3	p_3	Send the material from the warehouse.
y_4	p_5	Take material not meeting the requirements.
y_5	p_7	Adjust material to the demanded length.
y_6	p_8	Transport the prepared material to the saw station.
y_7	p_{10}	Bring the material to the operator.
y_8	p_{11}	Cut the material accordingly.
y_9	p_{12}	Put waste to the post-production waste storage.
y_{10}	p_{13}	Transport semi-finished product for milling.
y_{11}	p_{14}	Take the operator to the station.
y_{12}	p_{15}	Process material milling.
y_9	p_{16}	Put waste to the post-production waste storage.
y_{13}	p_{17}	Transfer to the quality check control.
y_{14}	p_{18}	Verify the quality.
y_{15}	p_{19}	Move to the packing station.
y_{16}	p_{20}	Make other post-production actions.
y_{17}	p_{21}	Pack the ready product.
y_{18}	p_{22}	Put into the warehouse.
y_{19}	p_{23}	End the manufacturing procedure.
y_{20}	p_{24}	Hand the finished product down to the customer.

Table 6.2: Description of input signals

Input Signal	Assigned to	Condition
x_1	t_1	Documentation is accepted.
x_2	t_2	Changes in documentation are made.
x_3	t_{13}	Quality requirements are not met.
x_4	t_{14}	Quality requirements are met.

model must therefore be improved. We add output transition t_{19} of p_{12} and p_{16} places, along with the transition output place p_{25} (marked in green), to the corrected specification shown in Fig. 6.2. They are responsible for waste storage and are synchronized with the main production process in transition t_{13}.

Subsequently, the new Petri net is reanalyzed by means of the 3rd proposed method. The algorithm finishes its operation with a result of potential coverage of the Petri net by place invariants. Then we use the 2nd proposed algorithm (q.v. Algorithm 4.2), which quickly finds the Petri net cover by p-invariants and confirms the boundedness of the net. If we know that the Petri net is covered by place invariants, then the 5th proposed method (q.v. Algorithm 4.5) is applied. It checks whether the obtained invariants form correct SM-components and whether they cover the Petri net. Also here we obtain an affirmative answer. The analysis is terminated and the Petri net is safe (1-bounded). The analysis

of the model was possible due to the usage of proposed methods in contrast to popular solutions, e.g. PIPE, used for instance, by designers of such manufacturing systems [163].

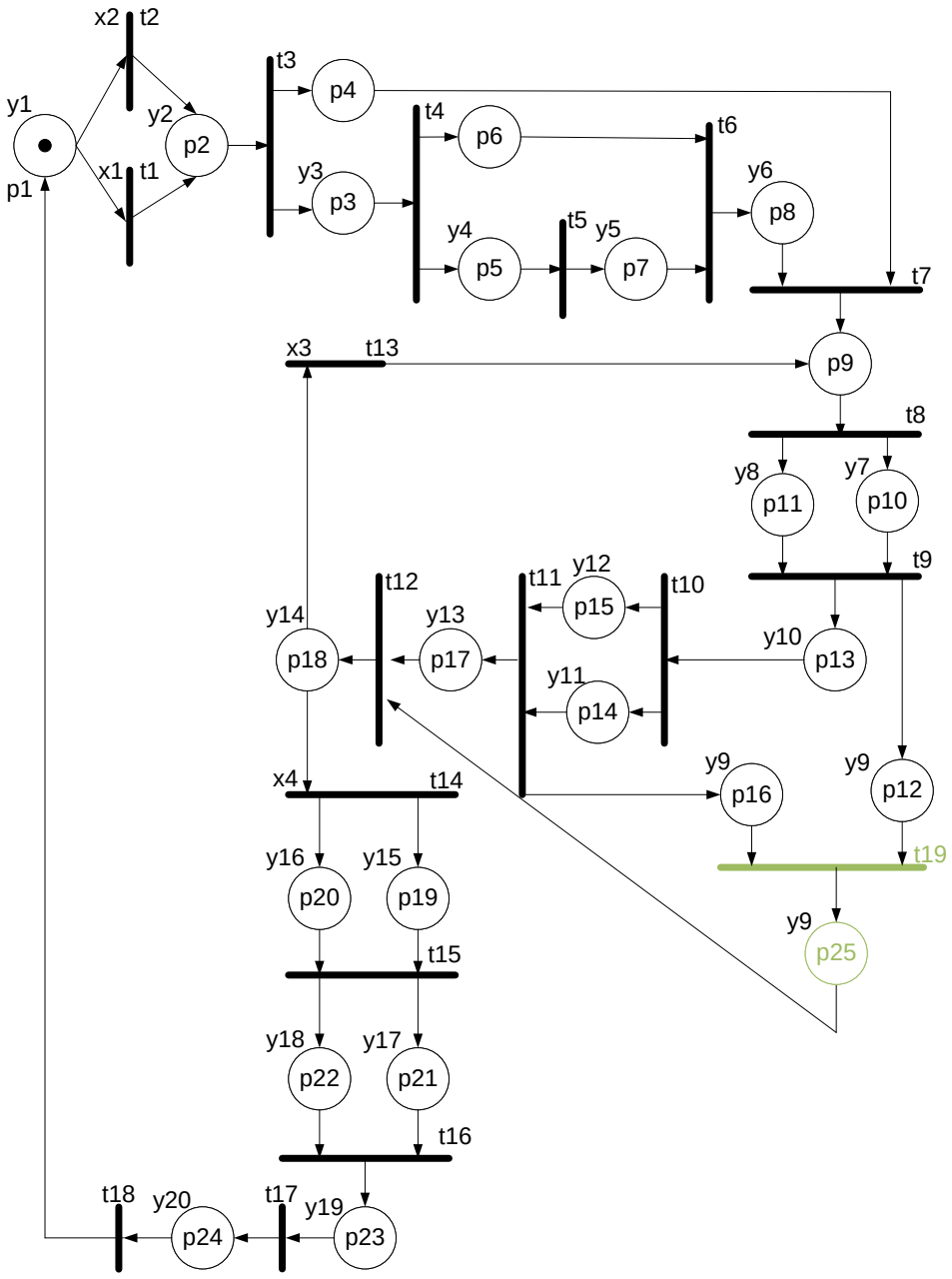


Figure 6.2: A real-life manufacturing system specified by a Petri net

Chapter 7

Conclusions

Petri nets have gained their popularity among researchers and engineers due to the combination of their graphic form of presentation with a mathematical description of operation and the possibility of modeling concurrent control systems. Based on mathematical formalism, it is possible to clearly determine the correctness and robustness of the modeled system, while graphical projecting of algorithms controlling control systems promotes the ease of the design process. In addition, it is possible to automatically generate codes ready for implementation in microprocessor or FPGA devices.

The gap in effective and efficient methods of analyzing boundedness and safeness, i.e. two key properties in modeling systems specified by Petri nets, constituted a chief motivation to undertake the research described in this dissertation. The analysis of the properties is not a trivial issue, as it involves exponential computational complexity in the general case. Therefore, the aim of this work was to propose novel methods of verification, dedicated separately to the efficiency or effectiveness criterion. The introduced algorithms were implemented and compared to known reference methods by several comprehensive tests. A library of 243 Petri nets of various levels of complexity from the Hippo project constituted a basis of the applied test modules. Full results of the performed experiments can be found in the Appendix A. The implemented novel algorithms have become part of the Hippo project developed at the University of Zielona Góra.

This chapter provides a synthetic summary of the research with the presentation of key innovative elements of the author's over four-year work. The chapter also comments on the thesis as well as on the contribution of this dissertation. Moreover, it outlines possible directions of further endeavors.

7.1 Confirmation of the Thesis

Both the successful implementation of the selected research goals and the presentation of the related tasks confirm the correctness of the thesis. The proposed methods based on reachability graphs are dedicated to effectiveness, while the algorithms based on the invariants cover are dedicated to efficiency. Moreover, another method based on a reduced row echelon form matrix makes it possible to estimate which algorithm to apply in a particular case of a Petri net. Combining it, the proposed method using Gauss-Jordan elimination was applied, along with novel methods dedicated to efficiency and effectiveness. This created a toolchain that allowed obtaining the desired result in the assumed time of less than one hour. Hence, it was possible to analyze all 243 benchmarks from the Hippo library effectively and efficiently.

7.2 Contribution Summary

The accomplishment of the doctoral thesis concluded with the development, implementation, and thorough testing of a total of five novel algorithms. All research results presented in this dissertation have been published in prestigious journals (from the Journal Citation Reports list) or in peer-reviewed materials of international conferences (indexed in Web of Science and Scopus). The publications of the obtained results, including the possibility of presentation at the doctoral international conference DoCEIS 2022 in Lisbon, was possible thanks to National Science Center, Poland, funding under the grant no. 2019/35/B/ST6/01683. The main contribution of this dissertation can be summarized as follows:

- development and implementation of a novel algorithm for boundedness analysis of a Petri net-based specification based on a reachability graph;
- development and implementation of a novel algorithm for boundedness analysis of a Petri net-based specification based on place invariants cover (the proposed algorithm is a significant improvement of existing solutions);
- development and implementation of a novel algorithm for boundedness analysis of a Petri net-based specification based on matrix transformations and a reduced row echelon form;
- development and implementation of a toolchain combining three novel algorithms for effective and efficient boundedness analysis;
- development and implementation of a novel algorithm for safeness analysis of a Petri net-based specification based on a reachability graph;

- development and implementation of a novel algorithm for safeness analysis of a Petri net-based specification based on state machine cover (the proposed algorithm is a significant improvement of existing solutions);
- verification of effectiveness and efficiency of the proposed methods compared to reference methods;
- discussion on conclusions of the experimental research;
- participation in the development of the Hippo system, support in the process of its conception and analysis of concurrent control systems specified by Petri nets.

7.3 Limitations and further Work

Although the proposed methods are becoming a comprehensive solution and can be used in practical applications, there are still possible further research directions, as the discussed problems, such as the construction of state space or the acquisition of all minimal invariants, are characterized by exponential computational complexity in the general case. In addition to the search for further improvements in the introduced algorithms, plans may include:

- the relationship between the lack of place invariants cover and the property of boundedness for the specification of concurrent control systems;
- broader application of the novel algorithm based on a matrix in a reduced row echelon form;
- analysis of other properties of Petri nets, such as liveness;
- development of tools supporting the design of concurrent control systems described by Petri nets.

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Appendix A

Detailed Experimental Results

This appendix includes full experimental verification of properties of boundedness and safeness.

A.1 Boundedness

Table A.1: Full results of experiments, state space-based algorithms

Name	$\|P\|$	$\|T\|$	Reference method		1st proposed method	
			bounded	runtime[ms]	bounded	runtime[ms]
prio_ex	2	3	true	56.8116	true	18.2667
semaphore	3	4	true	22.6309	true	19.2944
coloured	4	3	true	25.5637	true	17.2615
traffic_light_v2	4	3	true	18.1265	true	15.3904
pn_silva_05e	4	4	false	23.7932	false	16.9574
tank_heating	4	4	true	19.8070	true	18.3392
Entrance1	4	6	true	21.8608	true	15.8056
kovalyov92	4	6	true	20.1515	true	14.5507
return_books	4	6	true	22.5549	true	16.4531
pn_silva_05c	5	3	false	23.8781	false	16.9954
PUMA_unloading	5	3	true	57.5249	true	18.7439
consumerReachability	5	4	false	22.2833	false	21.4140
pn_silva_05b	5	4	false	33.7118	false	21.9474
pn_silva_05f	5	4	true	44462.0000	true	19.1288
pn_silva_04	5	6	false	84.3863	false	21.4031
invariants_exponent_3_2	6	2	true	20.4170	true	14.3074
np3	6	3	true	61.0780	true	18.5387
gals-example	6	4	true	58.9145	true	17.6626
lnet_p1n1	6	4	true	21.2580	true	15.8184
pcncp	6	4	true	23.5670	true	18.2273

pn_silva_02	6	4	true	55.8726	true	19.4230
PUMA_loading	6	4	true	75.6695	true	24.4826
RHINO_loading	6	4	true	59.6393	true	16.7076
RHINO_unloading	6	4	true	21.8541	true	18.7518
lnet_p1n2	6	5	true	23.0505	true	13.9244
lnet_p6n3	6	5	true	50.4137	true	15.4346
net1	6	5	true	62.4939	true	17.8283
traffic_lights	6	5	true	19.8527	true	14.4626
fig311_01	6	6	true	29.7733	true	19.0608
PNwD	6	6	true	76.1078	true	33.1066
pn_desel_03	6	7	true	32.4344	true	17.1911
communications_protocol	6	8	true	25.1123	true	16.3243
health_care_process	6	8	true	22.3302	true	14.3417
cncrr001	7	4	true	19.9029	true	16.0355
FMS_main_SIPN	7	4	true	23.7193	true	15.9449
agostini1	7	5	true	18.4699	true	17.2495
CNC_machine	7	5	true	21.2913	true	17.5161
transfer	7	5	true	24.6107	true	19.3973
voorhoeve3	7	5	true	18.0765	true	14.1707
net2	7	6	false	73.3893	false	26.4170
pnbrexpl_05	7	6	true	23.4051	true	20.0628
two_traffic_lights	7	6	true	18.2165	true	16.2687
pn_desel_01	7	7	true	63.6640	true	22.6454
silva1	7	7	true	49.9186	true	19.6985
traffic_light_v1	8	3	true	16.7578	true	16.0724
agerwala1	8	4	true	20.2320	true	16.3630
gaubert2	8	4	true	22.4147	true	15.9989
mixer_mod2	8	5	true	24.3639	true	21.2758
pn_desel_02	8	5	true	66.3045	true	22.9693
prod_cons	8	5	true	87.3341	true	36.9736
sub-task_of_PLT_and_PMN~	8	5	true	18.9115	true	15.4892
bridge	8	6	true	34.7380	true	21.3181
cp2	8	6	true	33.6237	true	18.8141
Exe5_split	8	6	true	61.8587	true	17.6638
img_280	8	6	true	31.9077	true	20.0653
lab5	8	6	true	26.3049	true	18.8704
TP5_I	8	6	true	19.2282	true	17.7237
bit_protocol	8	7	false	53.0126	false	27.6159
net4	8	7	true	25.4435	true	21.7310
silva14	8	7	true	61.6688	true	21.8435
elevator_2	8	8	true	26.6660	true	16.9500
girault8	8	8	true	91.0943	true	28.9878
net3	8	8	true	65.6040	true	26.5484
hard_case	9	3	true	20.7165	true	13.7562
invariants_exponent_3_3	9	3	true	21.6780	true	13.9898
bridge_semaphore	9	6	true	27.2764	true	22.8517
cortadella1	9	6	true	29.8670	true	17.4769
lnet_p1n4	9	6	true	31.9204	true	17.6487
multi_robot	9	6	true	36.1574	true	24.6878
oneWayTransmissionSystem	9	6	true	81.1547	true	25.1693

semaphore2	9	6	true	63.7779	true	18.6128
simple_production_system	9	6	true	73.2633	true	24.9712
lnet_p2n3	9	7	true	22.2145	true	16.9937
mutual_exclusion	9	7	true	69.1031	true	21.5975
miczulski1	9	8	true	67.0112	true	20.4400
pnbrexpl_03	9	8	true	28.3472	true	20.9835
reactor_small	9	8	true	63.2469	true	20.8127
pn_silva_01	9	9	true	23.4675	true	20.5181
medeiros1	9	13	true	70.0372	true	18.9670
vanDerAalst6	9	13	true	23.0026	true	17.6228
np5	10	5	true	25.4073	true	19.3974
ConsistentExampleMessageView	10	6	true	65.9098	true	31.7462
pn_silva_05d	10	6	true	62.7914	true	19.0724
exOR	10	7	true	21.0055	true	14.3336
girault4	10	7	true	41.7014	true	28.7987
speedway	10	7	true	25.8559	true	18.4950
barkaoui2	10	8	true	21.4339	true	16.9570
credit_procedure	10	8	true	87.0372	true	21.4479
lnet_p5n3	10	8	true	75.7265	true	22.5288
medeiros2	10	8	true	67.8827	true	31.7088
pascal_reach	10	8	true	68.6434	true	24.2104
silva8	10	8	true	22.0701	true	18.8941
jeng2	10	9	true	23.2713	true	17.0552
chrzastowski1	10	10	true	31.8218	true	27.5416
sivaraman1	10	10	true	70.8227	true	24.9032
franczok1	10	14	true	31.2875	true	18.2740
cncrr002	11	7	true	55.8600	true	46.5294
dataflow_computation	11	7	true	34.0984	true	20.9507
girault7	11	7	true	60.8647	true	18.1181
parallel_managing_automatic~	11	7	false	91.7534	false	25.7083
campos1	11	8	true	37.3537	true	32.7718
julvez_1	11	8	true	130.7313	true	49.3727
lnet_p5n1	11	9	true	92.4287	true	27.4539
pnbrexpl_06	11	9	true	70.7797	true	22.2867
esparza1	11	10	true	96.4341	true	36.0863
fernandez2	11	10	true	25.1146	true	18.2166
rejers2	11	10	true	71.2806	true	26.3428
bause1	11	11	true	30.3726	true	23.5162
oil_separator_cover_s_net_v3	11	11	true	63.2569	true	23.0211
pnbrexpl_07	11	12	true	93.1761	true	48.4954
barkaoui1	11	13	true	33.1407	true	26.2544
li3	11	14	true	34.6593	true	21.8737
invariants_exponent_3_4	12	4	true	25.3045	true	16.7715
girault1	12	6	true	29.1027	true	24.9127
real-life_system_smarthome~	12	7	true	43.6172	true	28.1617
3carros	12	8	true	34.2907	true	31.4738
four_philosophers	12	8	true	78.9335	true	22.6871
lnet_p10n1	12	9	true	115.2752	true	48.9349
machine_shop	12	9	true	7184.0000	true	6.1159
campos2	12	10	true	30.1517	true	25.1777

julvez_2	12	11	true	76.8923	true	18.8027
lnet_p6n1	12	11	true	85.7223	true	29.8793
bouillard1	12	12	true	47.7573	true	46.0473
li2	12	12	true	40.4389	true	23.8308
maruster3	12	13	true	66.5455	true	24.4633
ping1	12	14	true	80.2771	true	33.5221
vanDerAalst1	12	14	true	38.5573	true	28.9366
weijters1	12	14	true	36.1803	true	28.8203
Elevator01	12	17	true	115.6899	true	54.2002
lnet_p2n2	13	7	true	61.3678	true	18.5705
reiseg1	13	9	true	92.1241	true	31.0416
ExampleStateView	13	10	true	109.9310	true	48.3871
frame_manufact	13	10	true	88.5783	true	27.0384
holloway1	13	10	true	98.4501	true	40.5108
pnbrexpl_02	13	10	true	90.7901	true	35.7532
fernandez1	13	11	true	51.7186	true	18.3868
hee2	13	11	true	63.7722	true	20.3965
lnet_p5n2	13	11	true	91.5835	true	35.2337
pnbrexpl_08	13	11	true	76.6387	true	31.1327
pn_campos_silva2	13	12	true	126.0241	true	57.0109
pnbrexpl_15	13	12	true	102.2357	true	45.4118
silva4v2	13	12	true	111.8692	true	54.4255
voorhoeve1	13	12	true	47.9559	true	44.0637
board_game	13	13	false	21.4183	false	15.2853
hack1	13	13	true	150.5684	true	53.8082
pn_campos_silva7	13	13	true	91.1612	true	41.3765
pnbrexpl_04	13	13	true	97.9136	true	33.6487
silva7	13	13	true	82.0985	true	40.8650
maruster1	13	14	true	97.8088	true	32.3814
esparza3	13	15	true	110.6849	true	45.0946
girault3	14	9	true	67.2275	true	26.2178
philosophers_2	14	10	true	97.1781	true	37.8175
philosophers_2_rev_2	14	10	true	95.8451	true	38.9143
pn_silva_03	14	10	true	115.7270	true	52.3490
rejers1	14	11	true	101.2150	true	52.1882
hee1	14	12	true	81.6886	true	29.1998
voorhoeve2	14	12	true	55.5967	true	43.3879
pnbrexpl_12	14	13	true	95.2474	true	49.2921
vanDerAalst3	14	13	true	37.9714	true	29.2048
vanDerAalst7	14	13	true	55.7063	true	48.2878
jeng1	14	16	true	82.5513	true	32.6253
lnet_p6n2	14	16	false	1316.9184	false	49.1367
lnet_p9n1	14	16	true	119.1472	true	99.9595
desel1	15	9	true	378.7918	true	108.0438
dining_philosophers	15	10	false	124.8357	false	45.7244
campos3	15	11	true	60.7875	true	52.4055
oil_separator_cover_s_net_v2	15	11	true	64.4652	true	23.3883
IEC	15	12	true	44.8522	true	30.2662
esparza2	15	13	true	103.4161	true	45.9300
lasire1	15	13	true	34.1462	true	27.5741

lnet_p8n4	15	16	true	112.9582	true	63.7016
dongen1	15	17	true	43.5313	true	27.6424
2pusher	15	18	false	104.1100	false	68.5973
gaubert1	16	8	true	32.5419	true	22.1618
silva5	16	8	true	94.6754	true	38.8790
girault2	16	10	true	97.8294	true	31.0547
brenner1	16	12	true	51507.5962	true	49074.9953
beverage	16	13	true	54.3064	true	42.5259
beverage_production_part2	16	13	true	55.4875	true	41.2558
maruster4	16	13	true	165.2804	true	59.8852
mixer_one_cup	16	13	true	95.4623	true	54.5898
pnbrexpl_09	16	13	true	111.0386	true	53.8896
chiola1	16	15	true	32.3218	true	24.3016
state_space16	16	16	true	3208.8901	true	1802.1074
handling_of_incoming_order	17	14	true	86.4162	true	28.1294
eshuis1	17	15	true	76.8377	true	32.9730
automation3	17	16	true	66.7310	true	78.4346
s_net_frame_manufact_quality~	17	16	true	38.4612	true	31.0045
li1	17	17	true	113.4483	true	64.7912
pn_campos_silva6	18	11	true	2692509.0000	true	8.4306
barkaoui3	18	13	true	68.4313	true	50.8150
balduzzi1	18	16	true	31.6941	true	30.2047
vanDerAalst4	18	16	true	89.3019	true	79.2355
kavi1	18	20	true	352.2237	true	118.4768
PWM_patterns	19	11	true	89.2849	true	32.5728
hulgaard1	19	12	true	81.2282	true	26.0502
mixer	19	15	true	114.7682	true	56.8489
s_net_frame_manufact_quality~	19	18	true	52.6186	true	43.4010
maruster2	20	14	true	116.9708	true	60.1574
holliday1	20	15	true	519.5248	true	176.0140
philosophers_5	20	15	true	169.5814	true	81.3015
beverage_production	20	16	true	59.5669	true	44.9109
miling_machine	20	16	true	186.6802	true	68.1431
milling	20	16	true	203.7742	true	65.6276
mixer_mod1	20	16	true	111.4485	true	53.7371
dingle1	20	20	true	83.8090	true	29.0316
basile1	21	17	true	107.5966	true	86.9539
lnet_p8n3	21	17	true	207.1881	true	73.0312
fernandez3	21	20	true	90.4209	true	29.5087
pn_fernandez3	21	20	true	95.5674	true	36.9348
heiner1	22	20	true	283.3456	true	95.5728
chrzastowski2	22	21	true	46.9244	true	44.2883
vanDerAalst5	23	23	true	78.8984	true	55.8145
adam1	24	12	true	57.1502	true	55.1834
xie1	24	26	true	116.9825	true	107.4893
HAN	24	40	true	100185.7668	true	50454.5466
oil_separator_cover_alfa_net	27	21	true	98.8265	true	51.1180
lnet_p8n2	28	17	true	209.8969	true	80.9932
zuberek3	29	21	true	201.6953	true	179.4626
alfa_net_copy_milling_quality	29	25	false	67.7053	false	55.6947

ConsistentExample	29	25	true	180.9862	true	80.8641
oil_separator_cover_s_net	29	25	true	87.3408	true	56.2373
zuberek4	30	21	true	225.1568	true	194.4964
zuberek1	30	22	true	121.8420	true	107.7838
well_built_alfa_net_subprocess	30	25	true	141.6397	true	139.1226
alfa_net_copy_milling_subprocess	30	26	true	48.6994	true	42.9979
s_net_copy_milling_subprocess	31	28	true	154.7889	true	176.1151
crossroadSM_FPGA	32	12	true	79.0077	true	30.4632
alfa_net_copy_milling_synchr	32	27	true	192.1757	true	207.2063
s_net_copy_milling_quality	32	29	false	23633.6107	false	142.5313
state_space32	32	32	n/a	n/a	n/a	n/a
vanDerAalst8	34	27	true	139.9571	true	120.1210
lnet_p4n1	37	41	true	105.9426	true	39.7104
zuberek5	41	31	true	175.7267	true	154.3409
lnet_p7n1	41	32	true	269648.5325	true	129680.3014
state_space48	48	48	n/a	n/a	n/a	n/a
PWM_extended	49	31	true	1110.2623	true	293.2315
lnet_p8n1	51	40	true	42494.7059	true	22073.1653
cn_crr7	56	15	true	191885.1152	true	95667.5924
cn_crr10	80	21	n/a	n/a	n/a	n/a
cn_crr15	120	31	n/a	n/a	n/a	n/a
cn_crr25	200	51	n/a	n/a	n/a	n/a

Table A.2: Full results of experiments, linear algebra-based algorithms

Name	$\|P\|$	$\|T\|$	Reference method		2nd proposed method	
			covered	runtime[ms]	covered	runtime[ms]
prio_ex	2	3	true	0.022	true	0.087
semaphore	3	4	true	0.053	true	0.084
coloured	4	3	true	0.045	true	0.088
traffic_light_v2	4	3	true	0.031	true	0.087
pn_silva_05e	4	4	false	0.061	false	0.603
tank_heating	4	4	true	0.063	true	0.079
Entrance1	4	6	true	0.022	true	0.114
kovalyov92	4	6	true	0.024	true	0.086
return_books	4	6	true	0.021	true	0.092
pn_silva_05c	5	3	false	0.042	false	0.093
PUMA_unloading	5	3	true	0.080	true	0.082
consumerReachability	5	4	false	0.076	false	0.115
pn_silva_05b	5	4	false	0.061	false	0.099
pn_silva_05f	5	4	true	0.026	true	0.101
pn_silva_04	5	6	false	0.023	false	0.336
invariants_exponent_3_2	6	2	true	0.041	true	0.096
np3	6	3	true	0.074	true	0.119
gals-example	6	4	true	0.028	true	0.119
lnet_p1n1	6	4	true	0.034	true	0.145
pcncp	6	4	true	0.070	true	0.116
pn_silva_02	6	4	true	0.043	true	0.089
PUMA_loading	6	4	true	0.055	true	0.087

RHINO_loading	6	4	true	0.139	true	0.085
RHINO_unloading	6	4	true	0.070	true	0.091
lnet_p1n2	6	5	true	0.145	true	0.108
lnet_p6n3	6	5	true	0.049	true	0.103
net1	6	5	true	0.115	true	0.109
traffic_lights	6	5	true	0.084	true	0.134
fig311_01	6	6	true	0.030	true	0.141
PNwD	6	6	true	0.034	true	0.149
pn_desel_03	6	7	true	0.033	true	0.125
communications_protocol	6	8	true	0.027	true	0.126
health_care_process	6	8	true	0.059	true	0.176
cncrr001	7	4	true	0.040	true	0.187
FMS_main_SIPN	7	4	true	0.071	true	0.177
agostini1	7	5	true	0.042	true	0.121
CNC_machine	7	5	true	0.033	true	0.092
transfer	7	5	false	0.036	false	0.135
voorhoeve3	7	5	true	0.174	true	0.101
net2	7	6	false	0.052	false	0.102
pnbrexpl_05	7	6	true	0.051	true	0.305
two_traffic_lights	7	6	true	0.062	true	0.126
pn_desel_01	7	7	true	0.038	true	0.096
silva1	7	7	true	0.076	true	0.103
traffic_light_v1	8	3	true	0.064	true	2.752
agerwala1	8	4	true	0.041	true	0.167
gaubert2	8	4	true	0.058	true	0.168
mixer_mod2	8	5	true	0.038	true	7.387
pn_desel_02	8	5	true	0.041	true	0.132
prod_cons	8	5	true	0.040	true	0.139
sub-task_of_PLT_and_PMN~	8	5	true	0.037	true	0.143
bridge	8	6	true	0.059	true	0.100
cp2	8	6	false	0.073	false	0.139
Exe5_split	8	6	true	0.065	true	0.099
img_280	8	6	true	0.042	true	0.117
lab5	8	6	true	0.042	true	0.142
TP5_I	8	6	true	0.047	true	0.103
bit_protocol	8	7	false	0.040	false	0.240
net4	8	7	true	0.120	true	0.157
silva14	8	7	true	0.051	true	0.494
elevator_2	8	8	true	0.032	true	0.105
girault8	8	8	true	0.045	true	0.158
net3	8	8	true	0.040	true	0.160
hard_case	9	3	true	0.214	true	0.125
invariants_exponent_3_3	9	3	true	0.104	true	0.131
bridge_semaphore	9	6	true	0.069	true	0.212
cortadella1	9	6	true	0.065	true	0.124
lnet_p1n4	9	6	true	0.071	true	0.142
multi_robot	9	6	true	0.052	true	0.171
oneWayTransmissionSystem	9	6	true	0.146	true	0.174
semaphore2	9	6	true	0.084	true	0.138
simple_production_system	9	6	true	0.092	true	0.207

lnet_p2n3	9	7	true	0.153	true	0.130
mutual_exclusion	9	7	true	0.044	true	0.235
miczulski1	9	8	true	0.055	true	0.132
pnbrexpl_03	9	8	true	0.110	true	0.276
reactor_small	9	8	true	0.134	true	0.132
pn_silva_01	9	9	true	0.062	true	0.150
medeiros1	9	13	true	0.049	true	0.192
vanDerAalst6	9	13	true	0.058	true	0.516
np5	10	5	true	0.086	true	0.220
ConsistentExampleMessageView	10	6	true	0.132	true	0.221
pn_silva_05d	10	6	false	0.091	false	0.207
exOR	10	7	false	0.066	false	0.260
girault4	10	7	true	0.053	true	0.186
speedway	10	7	true	0.506	true	0.246
barkaoui2	10	8	true	0.064	true	0.225
credit_procedure	10	8	true	0.046	true	0.133
lnet_p5n3	10	8	true	0.223	true	0.142
medeiros2	10	8	true	0.067	true	0.251
pascal_reach	10	8	true	0.080	true	1.150
silva8	10	8	false	0.067	false	0.161
jeng2	10	9	true	0.080	true	0.164
chrzastowski1	10	10	true	0.045	true	0.242
sivaraman1	10	10	true	0.047	true	0.146
franczok1	10	14	true	0.151	true	0.193
cncrr002	11	7	true	0.075	true	0.259
dataflow_computation	11	7	true	0.066	true	0.235
girault7	11	7	true	0.107	true	0.174
parallel_managing_automatic_devices	11	7	false	0.077	false	0.209
campos1	11	8	true	2.458	true	0.335
julvez_1	11	8	true	0.066	true	0.257
lnet_p5n1	11	9	true	0.047	true	0.157
pnbrexpl_06	11	9	true	0.090	true	0.164
esparza1	11	10	true	0.055	true	0.324
fernandez2	11	10	true	0.056	true	0.207
rejers2	11	10	true	0.083	true	0.137
bause1	11	11	false	0.055	false	0.283
oil_separator_cover_s_net_v3	11	11	true	0.072	true	0.221
pnbrexpl_07	11	12	true	0.098	true	0.190
barkaoui1	11	13	true	0.063	true	0.243
li3	11	14	true	0.091	true	0.234
invariants_exponent_3_4	12	4	true	0.635	true	0.137
girault1	12	6	true	0.084	true	0.350
real-life_system_smarthome~	12	7	true	0.081	true	0.194
3carros	12	8	true	0.090	true	0.116
four_philosophers	12	8	true	0.092	true	0.319
lnet_p10n1	12	9	true	0.073	true	0.129
machine_shop	12	9	false	0.168	false	0.315
campos2	12	10	true	0.058	true	0.318
julvez_2	12	11	true	0.082	true	0.560
lnet_p6n1	12	11	true	0.074	true	0.160

bouillard1	12	12	true	0.077	true	0.246
li2	12	12	true	0.067	true	0.136
maruster3	12	13	true	0.153	true	0.164
ping1	12	14	true	0.085	true	0.259
vanDerAalst1	12	14	true	0.092	true	0.230
weijters1	12	14	true	0.171	true	0.507
Elevator01	12	17	false	0.087	false	1.108
lnet_p2n2	13	7	true	0.137	true	0.289
reiseg1	13	9	true	0.098	true	0.265
ExampleStateView	13	10	false	0.070	false	0.315
frame_manufact	13	10	true	0.185	true	0.204
holloway1	13	10	true	0.938	true	0.218
pnbrexpl_02	13	10	true	0.083	true	0.135
fernandez1	13	11	true	0.155	true	0.297
hee2	13	11	true	0.159	true	0.470
lnet_p5n2	13	11	true	0.219	true	0.172
pnbrexpl_08	13	11	true	0.096	true	0.174
pn_campos_silva2	13	12	true	0.078	true	4.127
pnbrexpl_15	13	12	true	0.274	true	0.247
silva4v2	13	12	true	0.073	true	0.421
voorhoeve1	13	12	true	0.211	true	0.282
board_game	13	13	false	0.104	false	0.237
hack1	13	13	true	0.100	true	0.826
pn_campos_silva7	13	13	true	0.372	true	0.316
pnbrexpl_04	13	13	true	0.131	true	0.281
silva7	13	13	true	0.116	true	0.313
maruster1	13	14	true	0.128	true	0.278
esparza3	13	15	true	0.134	true	0.385
girault3	14	9	true	0.319	true	0.946
philosophers_2	14	10	true	0.107	true	0.463
philosophers_2_rev_2	14	10	true	0.112	true	0.473
pn_silva_03	14	10	true	0.290	true	0.321
rejers1	14	11	true	0.358	true	0.134
hee1	14	12	true	0.475	true	0.567
voorhoeve2	14	12	true	0.429	true	0.321
pnbrexpl_12	14	13	true	0.090	true	0.248
vanDerAalst3	14	13	true	0.125	true	0.138
vanDerAalst7	14	13	true	0.067	true	0.566
jeng1	14	16	true	0.061	true	0.309
lnet_p6n2	14	16	false	0.085	false	0.240
lnet_p9n1	14	16	true	0.201	true	0.264
desel1	15	9	true	0.061	true	0.157
dining_philosophers	15	10	false	0.068	false	0.331
campos3	15	11	true	0.131	true	0.566
oil_separator_cover_s_net_v2	15	11	true	0.388	true	3.051
IEC	15	12	true	0.110	true	0.213
esparza2	15	13	true	2.277	true	0.544
lasire1	15	13	true	0.117	true	0.180
lnet_p8n4	15	16	true	0.081	true	0.398
dongen1	15	17	true	0.068	true	0.319

2pusher	15	18	false	0.109	false	1.688
gaubert1	16	8	true	0.260	true	0.490
silva5	16	8	true	1.443	true	0.546
girault2	16	10	true	0.170	true	0.537
brenner1	16	12	true	0.095	true	0.519
beverage	16	13	true	0.094	true	0.272
beverage_production_part2	16	13	true	0.085	true	0.226
maruster4	16	13	true	0.164	true	0.214
mixer_one_cup	16	13	true	0.078	true	10.047
pnbrexpl_09	16	13	true	0.229	true	0.259
chiola1	16	15	true	0.076	true	0.573
state_space16	16	16	true	0.091	true	2.167
handling_of_incoming_order	17	14	true	0.087	true	0.403
eshuis1	17	15	true	0.130	true	0.327
automation3	17	16	false	0.083	false	0.567
s_net_frame_manufact_quality_v1	17	16	true	0.127	true	3.829
li1	17	17	true	0.546	true	0.251
pn_campos_silva6	18	11	true	0.140	true	0.590
barkaoui3	18	13	true	0.094	true	0.777
balduzzi1	18	16	false	0.107	false	0.656
vanDerAalst4	18	16	true	0.148	true	0.343
kavi1	18	20	true	0.222	true	0.503
PWM_patterns	19	11	true	0.280	true	0.313
hulgaard1	19	12	true	11.863	true	0.973
mixer	19	15	true	0.106	true	0.320
s_net_frame_manufact_quality_v2~	19	18	true	0.129	true	0.333
maruster2	20	14	true	0.162	true	0.563
holliday1	20	15	true	0.274	true	0.631
philosophers_5	20	15	true	0.123	true	0.555
beverage_production	20	16	true	0.158	true	0.346
miling_machine	20	16	true	0.137	true	0.193
milling	20	16	true	0.160	true	0.187
mixer_mod1	20	16	true	0.138	true	0.371
dingle1	20	20	true	0.100	true	0.895
basile1	21	17	true	0.145	true	0.422
lnet_p8n3	21	17	true	0.172	true	0.328
fernandez3	21	20	true	0.225	true	0.674
pn_fernandez3	21	20	true	0.202	true	0.927
heiner1	22	20	true	0.138	true	0.900
chrzastowski2	22	21	true	0.105	true	0.223
vanDerAalst5	23	23	true	0.174	true	0.726
adam1	24	12	true	1.805	true	1.884
xie1	24	26	true	1.285	true	0.631
HAN	24	40	true	0.102	true	1.235
oil_separator_cover_alfa_net	27	21	true	1.229	true	0.468
lnet_p8n2	28	17	true	0.394	true	4.591
zuberek3	29	21	true	0.568	true	0.680
alfa_net_copy_milling_quality	29	25	false	0.378	false	0.571
ConsistentExample	29	25	false	56 220.831	false	1 197.142
oil_separator_cover_s_net	29	25	true	0.406	true	0.559

zuberek4	30	21	true	1.920	true	1.324
zuberek1	30	22	true	12.592	true	1.647
well_built_alfa_net_copy_subprocess	30	25	true	0.263	true	0.537
alfa_net_copy_milling_subprocess	30	26	false	0.365	false	0.587
s_net_copy_milling_subprocess	31	28	true	0.513	true	0.874
crossroadSM_FPGA	32	12	true	891 400.791	true	1.449
alfa_net_copy_milling_synchr	32	27	true	0.363	true	0.644
s_net_copy_milling_quality	32	29	false	0.438	false	0.640
state_space32	32	32	true	0.264	true	2.254
vanDerAalst8	34	27	true	0.414	true	4.785
lnet_p4n1	37	41	true	0.170	true	4.689
zuberek5	41	31	n/a	n/a	true	2.391
lnet_p7n1	41	32	true	0.631	true	1.439
state_space48	48	48	true	0.375	true	8.344
PWM_extended	49	31	true	2.392	true	17.842
lnet_p8n1	51	40	false	1.068	false	9.713
cn_crr7	56	15	n/a	n/a	true	3.832
cn_crr10	80	21	n/a	n/a	true	12.782
cn_crr15	120	31	n/a	n/a	true	37.731
cn_crr25	200	51	n/a	n/a	true	206.393

Table A.3: Full results of experiments, the reduced row echelon form-based algorithm

Name	$\|P\|$	$\|T\|$	3rd proposed method		
			covered	further analysis	runtime[ms]
prio_ex	2	3	possibly	2nd proposed method	0.003
semaphore	3	4	possibly	2nd proposed method	0.003
coloured	4	3	possibly	2nd proposed method	0.003
traffic_light_v2	4	3	possibly	2nd proposed method	0.003
pn_silva_05e	4	4	false	1st proposed method	0.003
tank_heating	4	4	possibly	2nd proposed method	0.003
Entrance1	4	6	possibly	2nd proposed method	0.004
kovalyov92	4	6	possibly	2nd proposed method	0.003
return_books	4	6	possibly	2nd proposed method	0.003
pn_silva_05c	5	3	false	1st proposed method	0.003
PUMA_unloading	5	3	possibly	2nd proposed method	0.003
consumerReachability	5	4	false	1st proposed method	0.003
pn_silva_05b	5	4	false	1st proposed method	0.003
pn_silva_05f	5	4	possibly	2nd proposed method	0.003
pn_silva_04	5	6	false	1st proposed method	0.003
invariants_exponent_3_2	6	2	possibly	2nd proposed method	0.003
np3	6	3	possibly	2nd proposed method	0.003
gals-example	6	4	possibly	2nd proposed method	0.005
lnet_p1n1	6	4	possibly	2nd proposed method	0.003
pcncp	6	4	possibly	2nd proposed method	0.003
pn_silva_02	6	4	possibly	2nd proposed method	0.004
PUMA_loading	6	4	possibly	2nd proposed method	0.003
RHINO_loading	6	4	possibly	2nd proposed method	0.003
RHINO_unloading	6	4	possibly	2nd proposed method	0.003

lnet_p1n2	6	5	possibly	2nd proposed method	0.003
lnet_p6n3	6	5	possibly	2nd proposed method	0.003
net1	6	5	possibly	2nd proposed method	0.003
traffic_lights	6	5	possibly	2nd proposed method	0.004
fig311_01	6	6	possibly	2nd proposed method	0.004
PNwD	6	6	possibly	2nd proposed method	0.003
pn_desel_03	6	7	possibly	2nd proposed method	0.004
communications_protocol	6	8	possibly	2nd proposed method	0.004
health_care_process	6	8	possibly	2nd proposed method	0.003
cncrr001	7	4	possibly	2nd proposed method	0.003
FMS_main_SIPN	7	4	possibly	2nd proposed method	0.003
agostini1	7	5	possibly	2nd proposed method	0.004
CNC_machine	7	5	possibly	2nd proposed method	0.004
transfer	7	5	possibly	1st proposed method	0.003
voorhoeve3	7	5	possibly	2nd proposed method	0.003
net2	7	6	false	1st proposed method	0.003
pnbrexpl_05	7	6	possibly	2nd proposed method	0.003
two_traffic_lights	7	6	possibly	2nd proposed method	0.003
pn_desel_01	7	7	possibly	2nd proposed method	0.004
silva1	7	7	possibly	2nd proposed method	0.004
traffic_light_v1	8	3	possibly	2nd proposed method	0.003
agerwala1	8	4	possibly	2nd proposed method	0.003
gaubert2	8	4	possibly	2nd proposed method	0.003
mixer_mod2	8	5	possibly	2nd proposed method	0.003
pn_desel_02	8	5	possibly	2nd proposed method	0.004
prod_cons	8	5	possibly	2nd proposed method	0.004
sub-task_of_PLT_and_PMN~	8	5	possibly	2nd proposed method	0.003
bridge	8	6	possibly	2nd proposed method	0.003
cp2	8	6	possibly	1st proposed method	0.003
Exe5_split	8	6	possibly	2nd proposed method	0.004
img_280	8	6	possibly	2nd proposed method	0.003
lab5	8	6	possibly	2nd proposed method	0.003
TP5_I	8	6	possibly	2nd proposed method	0.004
bit_protocol	8	7	false	1st proposed method	0.004
net4	8	7	possibly	2nd proposed method	0.003
silva14	8	7	possibly	2nd proposed method	0.004
elevator_2	8	8	possibly	2nd proposed method	0.004
girault8	8	8	possibly	2nd proposed method	0.004
net3	8	8	possibly	2nd proposed method	0.003
hard_case	9	3	possibly	2nd proposed method	0.003
invariants_exponent_3_3	9	3	possibly	2nd proposed method	0.003
bridge_semaphore	9	6	possibly	2nd proposed method	0.003
cortadella1	9	6	possibly	2nd proposed method	0.003
lnet_p1n4	9	6	possibly	2nd proposed method	0.004
multi_robot	9	6	possibly	2nd proposed method	0.004
oneWayTransmissionSystem	9	6	possibly	2nd proposed method	0.004
semaphore2	9	6	possibly	2nd proposed method	0.004
simple_production_system	9	6	possibly	2nd proposed method	0.004
lnet_p2n3	9	7	possibly	2nd proposed method	0.004
mutual_exclusion	9	7	possibly	2nd proposed method	0.004

miczulski1	9	8	possibly	2nd proposed method	0.004
pnbrexpl_03	9	8	possibly	2nd proposed method	0.004
reactor_small	9	8	possibly	2nd proposed method	0.004
pn_silva_01	9	9	possibly	2nd proposed method	0.004
medeiros1	9	13	possibly	2nd proposed method	0.005
vanDerAalst6	9	13	possibly	2nd proposed method	0.005
np5	10	5	possibly	2nd proposed method	0.003
ConsistentExampleMessageView	10	6	possibly	2nd proposed method	0.004
pn_silva_05d	10	6	possibly	1st proposed method	0.003
exOR	10	7	possibly	1st proposed method	0.005
girault4	10	7	possibly	2nd proposed method	0.004
speedway	10	7	possibly	2nd proposed method	0.004
barkaoui2	10	8	possibly	2nd proposed method	0.004
credit_procedure	10	8	possibly	2nd proposed method	0.005
lnet_p5n3	10	8	possibly	2nd proposed method	0.004
medeiros2	10	8	possibly	2nd proposed method	0.004
pascal_reach	10	8	possibly	2nd proposed method	0.004
silva8	10	8	possibly	1st proposed method	0.004
jeng2	10	9	possibly	2nd proposed method	0.005
chrzastowski1	10	10	possibly	2nd proposed method	0.005
sivaraman1	10	10	possibly	2nd proposed method	0.005
franczok1	10	14	possibly	2nd proposed method	0.006
cncrr002	11	7	possibly	2nd proposed method	0.004
dataflow_computation	11	7	possibly	2nd proposed method	0.004
girault7	11	7	possibly	2nd proposed method	0.004
parallel_managing_automatic	11	7	false	1st proposed method	0.004
campos1	11	8	possibly	2nd proposed method	0.004
julvez_1	11	8	possibly	2nd proposed method	0.004
lnet_p5n1	11	9	possibly	2nd proposed method	0.005
pnbrexpl_06	11	9	possibly	2nd proposed method	0.005
esparza1	11	10	possibly	2nd proposed method	0.005
fernandez2	11	10	possibly	2nd proposed method	0.006
rejers2	11	10	possibly	2nd proposed method	0.005
bause1	11	11	possibly	1st proposed method	0.005
oil_separator_cover_s_net_v3	11	11	possibly	2nd proposed method	0.005
pnbrexpl_07	11	12	possibly	2nd proposed method	0.005
barkaoui1	11	13	possibly	2nd proposed method	0.006
li3	11	14	possibly	2nd proposed method	0.007
invariants_exponent_3_4	12	4	possibly	2nd proposed method	0.004
girault1	12	6	possibly	2nd proposed method	0.004
real-life_system_smarthome~	12	7	possibly	2nd proposed method	0.004
3carros	12	8	possibly	2nd proposed method	0.005
four_philosophers	12	8	possibly	2nd proposed method	0.004
lnet_p10n1	12	9	possibly	2nd proposed method	0.005
machine_shop	12	9	possibly	1st proposed method	0.005
campos2	12	10	possibly	2nd proposed method	0.005
julvez_2	12	11	possibly	2nd proposed method	0.005
lnet_p6n1	12	11	possibly	2nd proposed method	0.005
bouillard1	12	12	possibly	2nd proposed method	0.006
li2	12	12	possibly	2nd proposed method	0.006

maruster3	12	13	possibly	2nd proposed method	0.006
ping1	12	14	possibly	2nd proposed method	0.007
vanDerAalst1	12	14	possibly	2nd proposed method	0.007
weijters1	12	14	possibly	2nd proposed method	0.007
Elevator01	12	17	possibly	1st proposed method	0.007
lnet_p2n2	13	7	possibly	2nd proposed method	0.004
reiseg1	13	9	possibly	2nd proposed method	0.005
ExampleStateView	13	10	possibly	1st proposed method	0.005
frame_manufact	13	10	possibly	2nd proposed method	0.006
holloway1	13	10	possibly	2nd proposed method	0.006
pnbrexpl_02	13	10	possibly	2nd proposed method	0.006
fernandez1	13	11	possibly	2nd proposed method	0.006
hee2	13	11	possibly	2nd proposed method	0.005
lnet_p5n2	13	11	possibly	2nd proposed method	0.007
pnbrexpl_08	13	11	possibly	2nd proposed method	0.006
pn_campos_silva2	13	12	possibly	2nd proposed method	0.006
pnbrexpl_15	13	12	possibly	2nd proposed method	0.006
silva4v2	13	12	possibly	2nd proposed method	0.006
voorhoeve1	13	12	possibly	2nd proposed method	0.006
board_game	13	13	false	1st proposed method	0.007
hack1	13	13	possibly	2nd proposed method	0.006
pn_campos_silva7	13	13	possibly	2nd proposed method	0.007
pnbrexpl_04	13	13	possibly	2nd proposed method	0.007
silva7	13	13	possibly	2nd proposed method	0.007
maruster1	13	14	possibly	2nd proposed method	0.007
esparza3	13	15	possibly	2nd proposed method	0.008
girault3	14	9	possibly	2nd proposed method	0.005
philosophers_2_rev_2	14	10	possibly	2nd proposed method	0.006
philosophers_2	14	10	possibly	2nd proposed method	0.006
pn_silva_03	14	10	possibly	2nd proposed method	0.005
rejers1	14	11	possibly	2nd proposed method	0.006
hee1	14	12	possibly	2nd proposed method	0.006
voorhoeve2	14	12	possibly	2nd proposed method	0.006
pnbrexpl_12	14	13	possibly	2nd proposed method	0.007
vanDerAalst3	14	13	possibly	2nd proposed method	0.008
vanDerAalst7	14	13	possibly	2nd proposed method	0.007
jeng1	14	16	possibly	2nd proposed method	0.009
lnet_p6n2	14	16	false	1st proposed method	0.008
lnet_p9n1	14	16	possibly	2nd proposed method	0.008
desel1	15	9	possibly	2nd proposed method	0.012
dining_philosophers	15	10	false	1st proposed method	0.006
campos3	15	11	possibly	2nd proposed method	0.006
oil_separator_cover_s_net_v2	15	11	possibly	2nd proposed method	0.006
IEC	15	12	possibly	2nd proposed method	0.007
esparza2	15	13	possibly	2nd proposed method	0.008
lasire1	15	13	possibly	2nd proposed method	0.007
lnet_p8n4	15	16	possibly	2nd proposed method	0.009
dongen1	15	17	possibly	2nd proposed method	0.010
2pusher	15	18	false	1st proposed method	0.009
gaubert1	16	8	possibly	2nd proposed method	0.005

silva5	16	8	possibly	2nd proposed method	0.005
girault2	16	10	possibly	2nd proposed method	0.007
brenner1	16	12	possibly	2nd proposed method	0.010
beverage_production_part2	16	13	possibly	2nd proposed method	0.009
beverage	16	13	possibly	2nd proposed method	0.008
maruster4	16	13	possibly	2nd proposed method	0.009
mixer_one_cup	16	13	possibly	2nd proposed method	0.008
pnbrexpl_09	16	13	possibly	2nd proposed method	0.008
chiola1	16	15	possibly	2nd proposed method	0.009
state_space16	16	16	possibly	2nd proposed method	0.008
handling_of_incoming_order	17	14	possibly	2nd proposed method	0.009
eshuis1	17	15	possibly	2nd proposed method	0.010
automation3	17	16	possibly	1st proposed method	0.010
s_net_frame_manufact	17	16	possibly	2nd proposed method	0.010
li1	17	17	possibly	2nd proposed method	0.011
pn_campos_silva6	18	11	possibly	2nd proposed method	0.007
barkaoui3	18	13	possibly	2nd proposed method	0.009
balduzzi1	18	16	possibly	1st proposed method	0.009
vanDerAalst4	18	16	possibly	2nd proposed method	0.011
kavi1	18	20	possibly	2nd proposed method	0.013
PWM_patterns	19	11	possibly	2nd proposed method	0.007
hulgaard1	19	12	possibly	2nd proposed method	0.008
mixer	19	15	possibly	2nd proposed method	0.011
s_net_frame_manufact	19	18	possibly	2nd proposed method	0.013
maruster2	20	14	possibly	2nd proposed method	0.010
holliday1	20	15	possibly	2nd proposed method	0.010
philosophers_5	20	15	possibly	2nd proposed method	0.009
beverage_production	20	16	possibly	2nd proposed method	0.013
miling_machine	20	16	possibly	2nd proposed method	0.013
milling	20	16	possibly	2nd proposed method	0.013
mixer_mod1	20	16	possibly	2nd proposed method	0.013
dingle1	20	20	possibly	2nd proposed method	0.015
basile1	21	17	possibly	2nd proposed method	0.013
lnet_p8n3	21	17	possibly	2nd proposed method	0.015
fernandez3	21	20	possibly	2nd proposed method	0.017
pn_fernandez3	21	20	possibly	2nd proposed method	0.016
heiner1	22	20	possibly	2nd proposed method	0.016
chrzastowski2	22	21	possibly	2nd proposed method	0.019
vanDerAalst5	23	23	possibly	2nd proposed method	0.021
adam1	24	12	possibly	2nd proposed method	0.008
xie1	24	26	possibly	2nd proposed method	0.025
HAN	24	40	possibly	2nd proposed method	0.035
oil_separator_cover_alfa_net	27	21	possibly	2nd proposed method	0.025
lnet_p8n2	28	17	possibly	2nd proposed method	0.018
zuberek3	29	21	possibly	2nd proposed method	0.025
alfa_net_copy_milling_quality	29	25	false	1st proposed method	0.030
oil_separator_cover_s_net	29	25	possibly	2nd proposed method	0.032
ConsistentExample	29	26	possibly	1st proposed method	0.042
zuberek4	30	21	possibly	2nd proposed method	0.027
zuberek1	30	22	possibly	2nd proposed method	0.028

well_built_alfa_net	30	25	possibly	2nd proposed method	0.035
alfa_net_copy_milling_subprocess	30	26	possibly	1st proposed method	0.035
s_net_copy_milling	31	28	possibly	2nd proposed method	0.042
crossroadSM_FPGA	32	12	possibly	2nd proposed method	0.012
alfa_net_copy_milling_synch	32	27	possibly	2nd proposed method	0.053
s_net_copy_milling	32	29	false	1st proposed method	0.052
state_space32	32	32	possibly	2nd proposed method	0.035
vanDerAalst8	34	27	possibly	2nd proposed method	0.042
lnet_p4n1	37	41	possibly	2nd proposed method	0.093
zuberek5	41	31	possibly	2nd proposed method	0.068
lnet_p7n1	41	32	possibly	2nd proposed method	0.072
state_space48	48	48	possibly	2nd proposed method	0.100
PWM_extended	49	31	possibly	2nd proposed method	0.080
lnet_p8n1	51	40	possibly	1st proposed method	0.131
cn_crr7	56	15	possibly	2nd proposed method	0.026
cn_crr10	80	21	possibly	2nd proposed method	0.064
cn_crr15	120	31	possibly	2nd proposed method	0.193
cn_crr25	200	51	possibly	2nd proposed method	0.832

Table A.4: Full results of experiments, combined proposed methods

Name	$\|P\|$	$\|T\|$	Toolchain		runtime[ms]
			bounded	applied methods	
prio_ex	2	3	true	3rd + 2nd prop. method	0.090
semaphore	3	4	true	3rd + 2nd prop. method	0.087
coloured	4	3	true	3rd + 2nd prop. method	0.091
traffic_light_v2	4	3	true	3rd + 2nd prop. method	0.090
pn_silva_05e	4	4	false	3rd + 1st prop. method	16.960
tank_heating	4	4	true	3rd + 2nd prop. method	0.082
Entrance1	4	6	true	3rd + 2nd prop. method	0.118
kovalyov92	4	6	true	3rd + 2nd prop. method	0.089
return_books	4	6	true	3rd + 2nd prop. method	0.095
pn_silva_05c	5	3	false	3rd + 1st prop. method	16.998
PUMA_unloading	5	3	true	3rd + 2nd prop. method	0.085
consumerReachability	5	4	false	3rd + 1st prop. method	21.417
pn_silva_05b	5	4	false	3rd + 1st prop. method	21.950
pn_silva_05f	5	4	true	3rd + 2nd prop. method	0.104
pn_silva_04	5	6	false	3rd + 1st prop. method	21.406
invariants_exponent_3_2	6	2	true	3rd + 2nd prop. method	0.099
np3	6	3	true	3rd + 2nd prop. method	0.122
gals-example	6	4	true	3rd + 2nd prop. method	0.124
lnet_p1n1	6	4	true	3rd + 2nd prop. method	0.148
pcncp	6	4	true	3rd + 2nd prop. method	0.119
pn_silva_02	6	4	true	3rd + 2nd prop. method	0.093
PUMA_loading	6	4	true	3rd + 2nd prop. method	0.090
RHINO_loading	6	4	true	3rd + 2nd prop. method	0.088
RHINO_unloading	6	4	true	3rd + 2nd prop. method	0.094
lnet_p1n2	6	5	true	3rd + 2nd prop. method	0.111
lnet_p6n3	6	5	true	3rd + 2nd prop. method	0.106
net1	6	5	true	3rd + 2nd prop. method	0.112

traffic_lights	6	5	true	3rd + 2nd prop. method	0.138
fig311_01	6	6	true	3rd + 2nd prop. method	0.145
PNwD	6	6	true	3rd + 2nd prop. method	0.152
pn_desel_03	6	7	true	3rd + 2nd prop. method	0.129
communications_protocol	6	8	true	3rd + 2nd prop. method	0.130
health_care_process	6	8	true	3rd + 2nd prop. method	0.179
cncrr001	7	4	true	3rd + 2nd prop. method	0.190
FMS_main_SIPN	7	4	true	3rd + 2nd prop. method	0.180
agostini1	7	5	true	3rd + 2nd prop. method	0.125
CNC_machine	7	5	true	3rd + 2nd prop. method	0.096
transfer	7	5	true	3rd + 1st prop. method	24.614
voorhoeve3	7	5	true	3rd + 2nd prop. method	0.104
net2	7	6	false	3rd + 1st prop. method	26.420
pnbrexpl_05	7	6	true	3rd + 2nd prop. method	0.308
two_traffic_lights	7	6	true	3rd + 2nd prop. method	0.129
pn_desel_01	7	7	true	3rd + 2nd prop. method	0.100
silva1	7	7	true	3rd + 2nd prop. method	0.107
traffic_light_v1	8	3	true	3rd + 2nd prop. method	2.755
agerwala1	8	4	true	3rd + 2nd prop. method	0.170
gaubert2	8	4	true	3rd + 2nd prop. method	0.171
mixer_mod2	8	5	true	3rd + 2nd prop. method	7.390
pn_desel_02	8	5	true	3rd + 2nd prop. method	0.136
prod_cons	8	5	true	3rd + 2nd prop. method	0.143
sub-task_of_PLT_and_PMN~	8	5	true	3rd + 2nd prop. method	0.146
bridge	8	6	true	3rd + 2nd prop. method	0.103
cp2	8	6	true	3rd + 1st prop. method	33.627
Exe5_split	8	6	true	3rd + 2nd prop. method	0.103
img_280	8	6	true	3rd + 2nd prop. method	0.120
lab5	8	6	true	3rd + 2nd prop. method	0.145
TP5_I	8	6	true	3rd + 2nd prop. method	0.107
bit_protocol	8	7	false	3rd + 1st prop. method	27.620
net4	8	7	true	3rd + 2nd prop. method	0.160
silva14	8	7	true	3rd + 2nd prop. method	0.498
elevator_2	8	8	true	3rd + 2nd prop. method	0.109
girault8	8	8	true	3rd + 2nd prop. method	0.162
net3	8	8	true	3rd + 2nd prop. method	0.163
hard_case	9	3	true	3rd + 2nd prop. method	0.128
invariants_exponent_3_3	9	3	true	3rd + 2nd prop. method	0.134
bridge_semaphore	9	6	true	3rd + 2nd prop. method	0.215
cortadella1	9	6	true	3rd + 2nd prop. method	0.127
lnet_p1n4	9	6	true	3rd + 2nd prop. method	0.146
multi_robot	9	6	true	3rd + 2nd prop. method	0.175
oneWayTransmissionSystem	9	6	true	3rd + 2nd prop. method	0.178
semaphore2	9	6	true	3rd + 2nd prop. method	0.142
simple_production_system	9	6	true	3rd + 2nd prop. method	0.211
lnet_p2n3	9	7	true	3rd + 2nd prop. method	0.134
mutual_exclusion	9	7	true	3rd + 2nd prop. method	0.239
miczulski1	9	8	true	3rd + 2nd prop. method	0.136
pnbrexpl_03	9	8	true	3rd + 2nd prop. method	0.280
reactor_small	9	8	true	3rd + 2nd prop. method	0.136

pn_silva_01	9	9	true	3rd + 2nd prop. method	0.154
medeiros1	9	13	true	3rd + 2nd prop. method	0.197
vanDerAalst6	9	13	true	3rd + 2nd prop. method	0.521
np5	10	5	true	3rd + 2nd prop. method	0.223
ConsistentExampleMessageView	10	6	true	3rd + 2nd prop. method	0.225
pn_silva_05d	10	6	true	3rd + 1st prop. method	62.794
exOR	10	7	true	3rd + 1st prop. method	21.011
girault4	10	7	true	3rd + 2nd prop. method	0.190
speedway	10	7	true	3rd + 2nd prop. method	0.250
barkaoui2	10	8	true	3rd + 2nd prop. method	0.229
credit_procedure	10	8	true	3rd + 2nd prop. method	0.138
lnet_p5n3	10	8	true	3rd + 2nd prop. method	0.146
medeiros2	10	8	true	3rd + 2nd prop. method	0.255
pascal_reach	10	8	true	3rd + 2nd prop. method	1.154
silva8	10	8	true	3rd + 1st prop. method	22.074
jeng2	10	9	true	3rd + 2nd prop. method	0.169
chrzastowski1	10	10	true	3rd + 2nd prop. method	0.247
sivaraman1	10	10	true	3rd + 2nd prop. method	0.151
franczok1	10	14	true	3rd + 2nd prop. method	0.199
cncrr002	11	7	true	3rd + 2nd prop. method	0.263
dataflow_computation	11	7	true	3rd + 2nd prop. method	0.239
girault7	11	7	true	3rd + 2nd prop. method	0.178
parallel_managing_automatic	11	7	false	3rd + 1st prop. method	25.712
campos1	11	8	true	3rd + 2nd prop. method	0.339
julvez_1	11	8	true	3rd + 2nd prop. method	0.261
lnet_p5n1	11	9	true	3rd + 2nd prop. method	0.162
pnbrexpl_06	11	9	true	3rd + 2nd prop. method	0.169
esparza1	11	10	true	3rd + 2nd prop. method	0.329
fernandez2	11	10	true	3rd + 2nd prop. method	0.213
rejers2	11	10	true	3rd + 2nd prop. method	0.142
bause1	11	11	true	3rd + 1st prop. method	30.378
oil_separator_cover_s_net_v3	11	11	true	3rd + 2nd prop. method	0.226
pnbrexpl_07	11	12	true	3rd + 2nd prop. method	0.195
barkaoui1	11	13	true	3rd + 2nd prop. method	0.249
li3	11	14	true	3rd + 2nd prop. method	0.241
invariants_exponent_3_4	12	4	true	3rd + 2nd prop. method	0.141
girault1	12	6	true	3rd + 2nd prop. method	0.354
real-life_system_smarthome~	12	7	true	3rd + 2nd prop. method	0.198
3carros	12	8	true	3rd + 2nd prop. method	0.121
four_philosophers	12	8	true	3rd + 2nd prop. method	0.323
lnet_p10n1	12	9	true	3rd + 2nd prop. method	0.134
machine_shop	12	9	true	3rd + 1st prop. method	7184.005
campos2	12	10	true	3rd + 2nd prop. method	0.323
julvez_2	12	11	true	3rd + 2nd prop. method	0.565
lnet_p6n1	12	11	true	3rd + 2nd prop. method	0.165
bouillard1	12	12	true	3rd + 2nd prop. method	0.252
li2	12	12	true	3rd + 2nd prop. method	0.142
maruster3	12	13	true	3rd + 2nd prop. method	0.170
ping1	12	14	true	3rd + 2nd prop. method	0.266
vanDerAalst1	12	14	true	3rd + 2nd prop. method	0.237

weijters1	12	14	true	3rd + 2nd prop. method	0.514
Elevator01	12	17	true	3rd + 1st prop. method	115.697
lnet_p2n2	13	7	true	3rd + 2nd prop. method	0.293
reiseg1	13	9	true	3rd + 2nd prop. method	0.270
ExampleStateView	13	10	true	3rd + 1st prop. method	109.936
frame_manufact	13	10	true	3rd + 2nd prop. method	0.210
holloway1	13	10	true	3rd + 2nd prop. method	0.224
pnbrexpl_02	13	10	true	3rd + 2nd prop. method	0.141
fernandez1	13	11	true	3rd + 2nd prop. method	0.303
hee2	13	11	true	3rd + 2nd prop. method	0.475
lnet_p5n2	13	11	true	3rd + 2nd prop. method	0.179
pnbrexpl_08	13	11	true	3rd + 2nd prop. method	0.180
pn_campos_silva2	13	12	true	3rd + 2nd prop. method	4.133
pnbrexpl_15	13	12	true	3rd + 2nd prop. method	0.253
silva4v2	13	12	true	3rd + 2nd prop. method	0.427
voorhoeve1	13	12	true	3rd + 2nd prop. method	0.288
board_game	13	13	false	3rd + 1st prop. method	15.292
hack1	13	13	true	3rd + 2nd prop. method	0.832
pn_campos_silva7	13	13	true	3rd + 2nd prop. method	0.323
pnbrexpl_04	13	13	true	3rd + 2nd prop. method	0.288
silva7	13	13	true	3rd + 2nd prop. method	0.320
maruster1	13	14	true	3rd + 2nd prop. method	0.285
esparza3	13	15	true	3rd + 2nd prop. method	0.393
girault3	14	9	true	3rd + 2nd prop. method	0.951
philosophers_2_rev_2	14	10	true	3rd + 2nd prop. method	0.469
philosophers_2	14	10	true	3rd + 2nd prop. method	0.479
pn_silva_03	14	10	true	3rd + 2nd prop. method	0.326
rejers1	14	11	true	3rd + 2nd prop. method	0.140
hee1	14	12	true	3rd + 2nd prop. method	0.573
voorhoeve2	14	12	true	3rd + 2nd prop. method	0.327
pnbrexpl_12	14	13	true	3rd + 2nd prop. method	0.255
vanDerAalst3	14	13	true	3rd + 2nd prop. method	0.146
vanDerAalst7	14	13	true	3rd + 2nd prop. method	0.573
jeng1	14	16	true	3rd + 2nd prop. method	0.318
lnet_p6n2	14	16	false	3rd + 1st prop. method	49.145
lnet_p9n1	14	16	true	3rd + 2nd prop. method	0.272
desel1	15	9	true	3rd + 2nd prop. method	0.169
dining_philosophers	15	10	false	3rd + 1st prop. method	45.730
campos3	15	11	true	3rd + 2nd prop. method	0.572
oil_separator_cover_s_net_v2	15	11	true	3rd + 2nd prop. method	3.057
IEC	15	12	true	3rd + 2nd prop. method	0.220
esparza2	15	13	true	3rd + 2nd prop. method	0.552
lasire1	15	13	true	3rd + 2nd prop. method	0.187
lnet_p8n4	15	16	true	3rd + 2nd prop. method	0.407
dongen1	15	17	true	3rd + 2nd prop. method	0.329
2pusher	15	18	false	3rd + 1st prop. method	68.606
gaubert1	16	8	true	3rd + 2nd prop. method	0.495
silva5	16	8	true	3rd + 2nd prop. method	0.551
girault2	16	10	true	3rd + 2nd prop. method	0.544
brenner1	16	12	true	3rd + 2nd prop. method	0.529

beverage_production_part2	16	13	true	3rd + 2nd prop. method	0.281
beverage	16	13	true	3rd + 2nd prop. method	0.234
maruster4	16	13	true	3rd + 2nd prop. method	0.223
mixer_one_cup	16	13	true	3rd + 2nd prop. method	10.055
pnbrexpl_09	16	13	true	3rd + 2nd prop. method	0.267
chiola1	16	15	true	3rd + 2nd prop. method	0.582
state_space16	16	16	true	3rd + 2nd prop. method	2.175
handling_of_incoming_order	17	14	true	3rd + 2nd prop. method	0.412
eshuis1	17	15	true	3rd + 2nd prop. method	0.337
automation3	17	16	true	3rd + 1st prop. method	66.741
s_net_frame_manufact	17	16	true	3rd + 2nd prop. method	3.839
li1	17	17	true	3rd + 2nd prop. method	0.262
pn_campos_silva6	18	11	true	3rd + 2nd prop. method	0.597
barkaoui3	18	13	true	3rd + 2nd prop. method	0.786
balduzzi1	18	16	true	3rd + 1st prop. method	31.703
vanDerAalst4	18	16	true	3rd + 2nd prop. method	0.354
kavi1	18	20	true	3rd + 2nd prop. method	0.516
PWM_patterns	19	11	true	3rd + 2nd prop. method	0.320
hulgaard1	19	12	true	3rd + 2nd prop. method	0.981
mixer	19	15	true	3rd + 2nd prop. method	0.331
s_net_frame_manufact	19	18	true	3rd + 2nd prop. method	0.346
maruster2	20	14	true	3rd + 2nd prop. method	0.573
holliday1	20	15	true	3rd + 2nd prop. method	0.641
philosophers_5	20	15	true	3rd + 2nd prop. method	0.564
beverage_production	20	16	true	3rd + 2nd prop. method	0.359
miling_machine	20	16	true	3rd + 2nd prop. method	0.206
milling	20	16	true	3rd + 2nd prop. method	0.200
mixer_mod1	20	16	true	3rd + 2nd prop. method	0.384
dingle1	20	20	true	3rd + 2nd prop. method	0.910
basile1	21	17	true	3rd + 2nd prop. method	0.435
lnet_p8n3	21	17	true	3rd + 2nd prop. method	0.343
fernandez3	21	20	true	3rd + 2nd prop. method	0.691
pn_fernandez3	21	20	true	3rd + 2nd prop. method	0.943
heiner1	22	20	true	3rd + 2nd prop. method	0.916
chrzastowski2	22	21	true	3rd + 2nd prop. method	0.242
vanDerAalst5	23	23	true	3rd + 2nd prop. method	0.747
adam1	24	12	true	3rd + 2nd prop. method	1.892
xie1	24	26	true	3rd + 2nd prop. method	0.656
HAN	24	40	true	3rd + 2nd prop. method	1.270
oil_separator_cover_alfa_net	27	21	true	3rd + 2nd prop. method	0.493
lnet_p8n2	28	17	true	3rd + 2nd prop. method	4.609
zuberek3	29	21	true	3rd + 2nd prop. method	0.705
alfa_net_copy_milling_quality	29	25	false	3rd + 1st prop. method	55.725
oil_separator_cover_s_net	29	25	true	3rd + 2nd prop. method	181.018
ConsistentExample	29	26	true	3rd + 1st prop. method	0.601
zuberek4	30	21	true	3rd + 2nd prop. method	1.351
zuberek1	30	22	true	3rd + 2nd prop. method	1.675
well_built_alfa_net	30	25	true	3rd + 2nd prop. method	0.572
alfa_net_copy_milling_subprocess	30	26	true	3rd + 1st prop. method	48.734
s_net_copy_milling	31	28	true	3rd + 2nd prop. method	0.916

crossroadSM_FPGA	32	12	true	3rd + 2nd prop. method	1.461
alfa_net_copy_milling_synch	32	27	true	3rd + 2nd prop. method	0.697
s_net_copy_milling	32	29	false	3rd + 1st prop. method	142.583
state_space32	32	32	true	3rd + 2nd prop. method	2.289
vanDerAalst8	34	27	true	3rd + 2nd prop. method	4.827
lnet_p4n1	37	41	true	3rd + 2nd prop. method	4.782
zuberek5	41	31	true	3rd + 2nd prop. method	2.459
lnet_p7n1	41	32	true	3rd + 2nd prop. method	1.511
state_space48	48	48	true	3rd + 2nd prop. method	8.444
PWM_extended	49	31	true	3rd + 2nd prop. method	17.922
lnet_p8n1	51	40	true	3rd + 1st prop. method	42494.837
cn_crr7	56	15	true	3rd + 2nd prop. method	3.858
cn_crr10	80	21	true	3rd + 2nd prop. method	12.846
cn_crr15	120	31	true	3rd + 2nd prop. method	37.924
cn_crr25	200	51	true	3rd + 2nd prop. method	207.225

A.2 Safeness

Table A.5: Full results of experiments, state space-based algorithms

Name	$\|P\|$	$\|T\|$	Reference method		4th proposed method	
			safe	runtime[ms]	safe	runtime[ms]
prio_ex	2	3	true	19.0780	true	15.5175
semaphore	3	4	true	19.0501	true	15.6182
coloured	4	3	true	25.1824	true	16.2993
traffic_light_v2	4	3	true	22.1925	true	15.9034
pn_silva_05e	4	4	false	22.9779	false	13.9798
tank_heating	4	4	true	19.9551	true	13.0749
Entrance1	4	6	true	23.2347	true	16.3483
kovalyov92	4	6	true	20.1264	true	15.3583
return_books	4	6	true	28.4905	true	14.1438
pn_silva_05c	5	3	false	21.3463	false	13.7149
PUMA_unloading	5	3	true	20.9865	true	18.0956
consumerReachability	5	4	false	26.4103	false	14.5373
pn_silva_05b	5	4	false	32.2826	false	15.6270
pn_silva_05f	5	4	true	23.3149	true	14.4269
pn_silva_04	5	6	false	41.1586	false	18.0003
invariants_exponent_3_2	6	2	true	24.2562	true	17.9393
np3	6	3	true	25.4518	true	16.1491
gals-example	6	4	true	28.6123	true	18.6427
lnet_p1n1	6	4	true	24.3566	true	15.1327
pcncp	6	4	true	21.9622	true	16.2125
pn_silva_02	6	4	true	19.6898	true	15.0080
PUMA_loading	6	4	true	20.7406	true	14.5736
RHINO_loading	6	4	true	22.6433	true	15.5960
RHINO_unloading	6	4	true	23.7006	true	14.7157
lnet_p1n2	6	5	true	22.7569	true	14.4633
lnet_p6n3	6	5	true	25.7643	true	15.7221

net1	6	5	true	29.2881	true	16.3986
traffic_lights	6	5	true	71.1782	true	14.8824
fig311_01	6	6	true	23.7371	true	20.7076
PNwD	6	6	false	44.1532	false	5.0541
pn_desel_03	6	7	true	23.9147	true	15.0002
communications_protocol	6	8	true	23.6780	true	15.5404
health_care_process	6	8	true	26.3826	true	31.2681
cncrr001	7	4	true	18.5514	true	16.3655
FMS_main_SIPN	7	4	true	24.6493	true	15.9028
agostini1	7	5	true	17.7040	true	14.1424
CNC_machine	7	5	true	17.4798	true	17.6813
transfer	7	5	false	71.8959	false	19.4545
voorhoeve3	7	5	true	23.5822	true	19.2303
net2	7	6	false	43.1173	false	22.8879
pnbrexpl_05	7	6	true	22.7454	true	16.0845
two_traffic_lights	7	6	true	25.3653	true	15.7590
pn_desel_01	7	7	true	24.8940	true	18.4418
silva1	7	7	true	23.6710	true	17.2683
traffic_light_v1	8	3	true	21.4085	true	16.6392
agerwala1	8	4	true	16.4393	true	15.5919
gaubert2	8	4	true	25.4083	true	15.6056
mixer_mod2	8	5	true	24.4189	true	16.0223
pn_desel_02	8	5	true	30.4172	true	18.0021
prod_cons	8	5	true	35.2817	true	25.1723
sub-task_of_PLT_and_PMN˜	8	5	true	54.7568	true	15.1413
bridge	8	6	true	24.2906	true	18.2040
cp2	8	6	true	34.2382	true	19.6579
Exe5_split	8	6	true	23.0403	true	18.7990
img_280	8	6	true	36.9830	true	21.2906
lab5	8	6	true	25.9867	true	17.9450
TP5_I	8	6	true	64.7127	true	17.1282
bit_protocol	8	7	false	48.2556	false	27.0238
net4	8	7	true	26.2908	true	24.8691
silva14	8	7	true	27.5944	true	18.0024
elevator_2	8	8	true	27.9761	true	18.3702
girault8	8	8	true	43.4841	true	30.9473
net3	8	8	true	30.9293	true	17.4025
hard_case	9	3	true	24.4431	true	35.6013
invariants_exponent_3_3	9	3	true	24.4752	true	16.8175
bridge_semaphore	9	6	true	24.4369	true	17.8110
cortadella1	9	6	true	24.9902	true	18.1420
lnet_p1n4	9	6	true	24.1594	true	17.2365
multi_robot	9	6	true	29.1748	true	21.1245
oneWayTransmissionSystem	9	6	true	35.3004	true	21.5588
semaphore2	9	6	true	23.8924	true	16.7402
simple_production_system	9	6	true	33.8013	true	20.7964
lnet_p2n3	9	7	true	23.5703	true	17.1914
mutual_exclusion	9	7	true	33.7939	true	18.2336
miczulski1	9	8	true	27.4469	true	19.2999
pnbrexpl_03	9	8	true	28.9208	true	18.2586

reactor_small	9	8	true	28.2865	true	17.7473
pn_silva_01	9	9	true	23.0759	true	15.3197
medeiros1	9	13	true	27.5173	true	16.6554
vanDerAalst6	9	13	true	61.7860	true	16.8938
np5	10	5	true	21.8705	true	16.6962
ConsistentExampleMessageView	10	6	true	27.3633	true	19.9430
pn_silva_05d	10	6	true	23.3053	true	16.8381
exOR	10	7	true	19.8099	true	1157425.0000
girault4	10	7	true	42.2787	true	29.8964
speedway	10	7	true	24.4475	true	16.2454
barkaoui2	10	8	true	19.3042	true	16.5852
credit_procedure	10	8	true	37.2567	true	22.0580
lnet_p5n3	10	8	true	38.5621	true	23.7509
medeiros2	10	8	true	29.7437	true	19.3634
pascal_reach	10	8	true	33.7933	true	19.1851
silva8	10	8	true	23.9648	true	12.8131
jeng2	10	9	true	28.1174	true	21.5514
chrzastowski1	10	10	true	28.6589	true	29.6647
sivaraman1	10	10	true	30.2501	true	20.3373
franczok1	10	14	true	32.7726	true	18.3673
cncrr002	11	7	true	44.9520	true	43.9512
dataflow_computation	11	7	true	37.5029	true	23.9803
girault7	11	7	true	28.7862	true	22.5678
parallel_managing_automatic~	11	7	false	43.6825	false	23.1102
campos1	11	8	true	34.0170	true	49.6388
julvez_1	11	8	true	76.1174	true	52.6802
lnet_p5n1	11	9	true	45.7711	true	26.0459
pnbrexpl_06	11	9	true	29.7195	true	18.2188
esparza1	11	10	true	53.2915	true	37.2374
fernandez2	11	10	true	25.2160	true	19.1360
rejers2	11	10	true	26.6619	true	19.8361
bause1	11	11	true	28.1506	true	23.5252
oil_separator_cover_s_net_v3	11	11	true	31.3567	true	21.7072
pnbrexpl_07	11	12	true	62.2016	true	42.2002
barkaoui1	11	13	true	31.7694	true	27.0118
li3	11	14	true	38.1803	true	24.0387
invariants_exponent_3_4	12	4	true	21.8958	true	16.5507
girault1	12	6	true	40.1864	true	18.5023
real-life_system_smarthome~	12	7	true	37.3951	true	24.4497
3carros	12	8	true	27.9686	true	29.1537
four_philosophers	12	8	true	32.9398	true	23.0046
lnet_p10n1	12	9	true	78.4702	true	48.0305
machine_shop	12	9	true	1153650.0000	true	5.1309
campos2	12	10	true	24.6993	true	24.3451
julvez_2	12	11	true	34.7127	true	18.5087
lnet_p6n1	12	11	true	48.5221	true	29.8166
bouillard1	12	12	true	49.1283	true	40.0612
li2	12	12	true	43.0491	true	27.1457
maruster3	12	13	true	28.5800	true	19.7536
ping1	12	14	true	35.1854	true	27.3093

vanDerAalst1	12	14	true	79.9651	true	26.9047
weijters1	12	14	true	80.0302	true	59.1239
Elevator01	12	17	true	75.6675	true	64.3786
lnet_p2n2	13	7	true	26.7647	true	21.5208
reiseg1	13	9	true	42.2186	true	26.8663
ExampleStateView	13	10	false	78.4403	false	44.1234
frame_manufact	13	10	true	43.2207	true	28.1516
holloway1	13	10	true	59.2947	true	43.6527
pnbrexpl_02	13	10	true	45.9803	true	30.7972
fernandez1	13	11	true	25.3959	true	18.8408
hee2	13	11	true	30.6845	true	37.5002
lnet_p5n2	13	11	true	54.1923	true	35.0413
pnbrexpl_08	13	11	true	38.3248	true	24.6866
pn_campos_silva2	13	12	true	72.3560	true	51.4288
pnbrexpl_15	13	12	true	50.0065	true	40.8692
silva4v2	13	12	true	69.5506	true	52.6372
voorhoeve1	13	12	true	112.3877	true	44.4445
board_game	13	13	false	21.7698	false	14.3919
hack1	13	13	true	66.4891	true	53.4076
pn_campos_silva7	13	13	true	45.6876	true	32.2327
pnbrexpl_04	13	13	true	43.5901	true	31.0588
silva7	13	13	true	44.8462	true	31.5231
maruster1	13	14	true	43.7007	true	29.5956
esparza3	13	15	true	68.2389	true	51.5460
girault3	14	9	true	38.2862	true	25.9363
philosophers_2	14	10	true	45.2491	true	34.3534
philosophers_2_rev_2	14	10	true	48.3218	true	35.4309
pn_silva_03	14	10	true	60.4660	true	42.1453
rejers1	14	11	true	74.0095	true	47.3889
hee1	14	12	true	40.9226	true	56.9268
voorhoeve2	14	12	true	101.0768	true	57.8930
pnbrexpl_12	14	13	true	50.3703	true	43.1007
vanDerAalst3	14	13	true	82.1756	true	29.7832
vanDerAalst7	14	13	true	122.2586	true	46.1532
jeng1	14	16	true	47.0792	true	34.6446
lnet_p6n2	14	16	false	712.1394	false	63.5090
lnet_p9n1	14	16	true	63.3058	true	46.1819
desel1	15	9	true	185.3470	true	127.7411
dining_philosophers	15	10	false	81.4642	false	48.5635
campos3	15	11	true	62.2191	true	53.1786
oil_separator_cover_s_net_v2	15	11	true	31.7127	true	19.6126
IEC	15	12	true	50.9404	true	32.4745
esparza2	15	13	true	58.5960	true	41.0707
lasire1	15	13	true	41.4052	true	21.0735
lnet_p8n4	15	16	true	75.1005	true	63.8796
dongen1	15	17	true	40.4876	true	32.7116
2pusher	15	18	false	111.1739	false	60.3173
gaubert1	16	8	true	34.4179	true	20.2161
silva5	16	8	true	52.8754	true	36.1776
girault2	16	10	true	49.6675	true	31.3195

brenner1	16	12	false	50702.5932	false	1047.3681
beverage	16	13	true	50.7734	true	44.4248
beverage_production_part2	16	13	true	48.5905	true	42.0991
maruster4	16	13	true	71.1404	true	54.7766
mixer_one_cup	16	13	true	66.9388	true	42.0077
pnbrexpl_09	16	13	true	64.9904	true	42.7809
chiola1	16	15	true	34.2005	true	27.5894
state_space16	16	16	true	1493.9547	true	1435.8302
handling_of_incoming_order	17	14	true	42.2454	true	29.6139
eshuis1	17	15	true	50.8159	true	30.3682
automation3	17	16	true	60.3088	true	72.3349
s_net_frame_manufact_quality~	17	16	true	91.6302	true	29.2645
li1	17	17	true	57.5485	true	44.2736
pn_campos_silva6	18	11	true	9.0868	true	4.9494
barkaoui3	18	13	true	63.8518	true	49.5238
balduzzi1	18	16	true	27.0226	true	23.2883
vanDerAalst4	18	16	true	240.5476	true	77.2516
kavi1	18	20	true	169.3538	true	123.0547
PWM_patterns	19	11	true	45.1191	true	28.2835
hulgaard1	19	12	true	41.0423	true	27.2892
mixer	19	15	true	63.0277	true	45.5941
s_net_frame_manufact_quality~	19	18	true	95.0531	true	41.6006
maruster2	20	14	true	65.5881	true	51.6015
holliday1	20	15	true	295.4204	true	183.8872
philosophers_5	20	15	true	105.0783	true	75.8251
beverage_production	20	16	true	57.7263	true	46.4452
miling_machine	20	16	true	89.0111	true	58.0210
milling	20	16	true	78.1119	true	60.3329
mixer_mod1	20	16	true	65.7985	true	46.1381
dingle1	20	20	true	45.4625	true	30.8079
basile1	21	17	true	106.6939	true	86.7165
lnet_p8n3	21	17	true	94.4362	true	67.8008
fernandez3	21	20	true	43.3925	true	30.9028
pn_fernandez3	21	20	true	48.2607	true	27.8295
heiner1	22	20	true	130.7727	true	128.0666
chrzastowski2	22	21	true	43.6154	true	41.7286
vanDerAalst5	23	23	true	115.9249	true	57.5321
adam1	24	12	true	54.2918	true	56.6285
xie1	24	26	true	292.7697	true	103.4013
HAN	24	40	true	50231.3384	true	53669.2779
oil_separator_cover_alfa_net	27	21	true	57.0549	true	41.8627
lnet_p8n2	28	17	true	99.5833	true	71.4567
zuberek3	29	21	true	692.0424	true	188.9980
alfa_net_copy_milling_quality	29	25	false	78.3404	false	50.3740
ConsistentExample	29	25	false	91.6349	false	45.2747
oil_separator_cover_s_net	29	25	true	66.2475	true	82.6780
zuberek4	30	21	true	631.3786	true	192.1192
zuberek1	30	22	true	281.9953	true	102.7223
well_built_alfa_net_subprocess	30	25	true	531.8516	true	141.2623
alfa_net_copy_subprocess	30	26	true	49.3245	true	43.1146

s_net_copy_milling_subprocess	31	28	true	433.4854	true	183.4345
crossroadSM_FPGA	32	12	true	43.8308	true	46.3888
alfa_net_copy_milling_synchr	32	27	true	238.9907	true	173.8360
s_net_copy_milling_quality	32	29	false	41407.1404	false	145.0464
state_space32	32	32	n/a	n/a	n/a	n/a
vanDerAalst8	34	27	true	352.7851	true	119.9650
lnet_p4n1	37	41	true	56.5367	true	48.7006
zuberek5	41	31	true	557.5064	true	146.1737
lnet_p7n1	41	32	true	127849.4493	true	132817.9162
state_space48	48	48	n/a	n/a	n/a	n/a
PWM_extended	49	31	true	398.9832	true	274.9435
lnet_p8n1	51	40	true	20157.0578	true	22438.7556
cn_crr7	56	15	true	93697.7373	true	96224.1716
cn_crr10	80	21	n/a	n/a	n/a	n/a
cn_crr15	120	31	n/a	n/a	n/a	n/a
cn_crr25	200	51	n/a	n/a	n/a	n/a

Table A.6: Full results of experiments, linear algebra-based algorithms

Name	$\|P\|$	$\|T\|$	Reference method		5th proposed method	
			covered	runtime[ms]	covered	runtime[ms]
prio_ex	2	3	true	0.025	true	0.639
semaphore	3	4	true	0.059	true	0.168
coloured	4	3	true	0.050	true	0.094
traffic_light_v2	4	3	true	0.033	true	0.088
pn_silva_05e	4	4	false	0.067	false	0.090
tank_heating	4	4	true	0.069	true	0.099
Entrance1	4	6	true	0.024	true	0.127
kovalyov92	4	6	true	0.026	true	0.096
return_books	4	6	true	0.023	true	0.116
pn_silva_05c	5	3	false	0.045	false	0.101
PUMA_unloading	5	3	true	0.088	true	0.083
consumerReachability	5	4	false	0.081	false	0.113
pn_silva_05b	5	4	false	0.066	false	0.121
pn_silva_05f	5	4	true	0.028	true	0.115
pn_silva_04	5	6	false	0.026	false	0.120
invariants_exponent_3_2	6	2	true	0.044	true	0.090
np3	6	3	true	0.080	true	0.115
gals-example	6	4	true	0.030	true	0.141
lnet_p1n1	6	4	true	0.036	true	0.120
pcncp	6	4	true	0.076	true	0.167
pn_silva_02	6	4	true	0.046	true	0.093
PUMA_loading	6	4	true	0.059	true	0.242
RHINO_loading	6	4	true	0.142	true	0.086
RHINO_unloading	6	4	true	0.073	true	0.199
lnet_p1n2	6	5	true	0.152	true	0.118
lnet_p6n3	6	5	true	0.052	true	0.107
net1	6	5	true	0.120	true	0.123
traffic_lights	6	5	true	0.090	true	0.151

fig311_01	6	6	true	0.032	true	0.134
PNwD	6	6	false	0.036	false	0.112
pn_desel_03	6	7	true	0.035	true	0.128
communications_protocol	6	8	true	0.029	true	0.132
health_care_process	6	8	true	0.063	true	0.178
cncrr001	7	4	true	0.043	true	0.165
FMS_main_SIPN	7	4	true	0.076	true	0.131
agostini1	7	5	true	0.047	true	0.109
CNC_machine	7	5	true	0.035	true	0.086
transfer	7	5	false	0.039	false	2.895
voorhoeve3	7	5	true	0.186	true	0.097
net2	7	6	false	0.056	false	0.110
pnbrexpl_05	7	6	true	0.068	true	0.253
two_traffic_lights	7	6	true	0.066	true	0.136
pn_desel_01	7	7	true	0.041	true	0.107
silva1	7	7	true	0.088	true	0.107
traffic_light_v1	8	3	true	0.068	true	0.187
agerwala1	8	4	true	0.044	true	0.229
gaubert2	8	4	true	0.062	true	0.206
mixer_mod2	8	5	true	0.040	true	0.226
pn_desel_02	8	5	true	0.044	true	0.135
prod_cons	8	5	true	0.042	true	0.148
sub-task_of_PLT_and_PMN~	8	5	true	0.041	true	0.152
bridge	8	6	true	0.062	true	0.104
cp2	8	6	false	0.080	false	0.162
Exe5_split	8	6	true	0.069	true	0.227
img_280	8	6	true	0.044	true	0.127
lab5	8	6	true	0.045	true	0.133
TP5_I	8	6	true	0.049	true	0.100
bit_protocol	8	7	false	0.042	false	0.178
net4	8	7	true	0.125	true	0.279
silva14	8	7	true	0.054	true	0.136
elevator_2	8	8	true	0.034	true	0.106
girault8	8	8	true	0.047	true	0.181
net3	8	8	true	0.042	true	0.133
hard_case	9	3	true	0.225	true	0.138
invariants_exponent_3_3	9	3	true	0.111	true	0.140
bridge_semaphore	9	6	true	0.072	true	0.229
cortadella1	9	6	true	0.068	true	0.170
lnet_p1n4	9	6	true	0.075	true	0.152
multi_robot	9	6	true	0.055	true	0.181
oneWayTransmissionSystem	9	6	true	0.153	true	0.178
semaphore2	9	6	true	0.089	true	0.161
simple_production_system	9	6	true	0.096	true	0.179
lnet_p2n3	9	7	true	0.211	true	0.134
mutual_exclusion	9	7	true	0.047	true	0.220
miczulski1	9	8	true	0.058	true	0.138
pnbrexpl_03	9	8	true	0.115	true	0.148
reactor_small	9	8	true	0.139	true	0.147
pn_silva_01	9	9	true	0.066	true	0.166

medeiros1	9	13	true	0.051	true	0.218
vanDerAalst6	9	13	true	0.060	true	0.214
np5	10	5	true	0.089	true	0.675
ConsistentExampleMessageView	10	6	true	0.137	true	0.271
pn_silva_05d	10	6	false	0.094	false	0.263
exOR	10	7	false	0.068	false	0.289
girault4	10	7	true	0.056	true	0.220
speedway	10	7	true	0.509	true	0.204
barkaoui2	10	8	true	0.067	true	0.233
credit_procedure	10	8	true	0.049	true	0.213
lnet_p5n3	10	8	true	0.231	true	0.147
medeiros2	10	8	true	0.069	true	0.192
pascal_reach	10	8	true	0.084	true	0.251
silva8	10	8	false	0.070	false	0.197
jeng2	10	9	true	0.084	true	0.177
chrzastowski1	10	10	true	0.048	true	0.216
sivaraman1	10	10	true	0.049	true	0.157
franczok1	10	14	true	0.157	true	0.203
cncrr002	11	7	true	0.078	true	0.273
dataflow_computation	11	7	true	0.069	true	0.244
girault7	11	7	true	0.116	true	0.165
parallel_managing_automatic_devices	11	7	false	0.080	false	0.232
campos1	11	8	true	2.461	true	0.269
julvez_1	11	8	true	0.068	true	0.231
lnet_p5n1	11	9	true	0.055	true	0.188
pnbrexpl_06	11	9	true	0.093	true	0.133
esparza1	11	10	true	0.057	true	0.359
fernandez2	11	10	true	0.059	true	0.238
rejers2	11	10	true	0.087	true	0.144
bause1	11	11	false	0.058	false	0.277
oil_separator_cover_s_net_v3	11	11	true	0.075	true	0.232
pnbrexpl_07	11	12	true	0.103	true	0.192
barkaoui1	11	13	true	0.065	true	0.237
li3	11	14	true	0.093	true	0.231
invariants_exponent_3_4	12	4	true	0.671	true	0.183
girault1	12	6	true	0.094	true	0.347
real-life_system_smarthome~	12	7	true	0.094	true	0.186
3carros	12	8	true	0.094	true	0.179
four_philosophers	12	8	true	0.097	true	0.277
lnet_p10n1	12	9	true	0.075	true	0.225
machine_shop	12	9	false	0.175	false	0.333
campos2	12	10	true	0.068	true	0.360
julvez_2	12	11	true	0.085	true	0.224
lnet_p6n1	12	11	true	0.077	true	0.166
bouillard1	12	12	true	0.081	true	0.284
li2	12	12	true	0.069	true	0.142
maruster3	12	13	true	0.156	true	0.184
ping1	12	14	true	0.088	true	0.279
vanDerAalst1	12	14	true	0.095	true	0.292
weijters1	12	14	true	0.173	true	0.279

Elevator01	12	17	false	0.090	false	0.386
lnet_p2n2	13	7	true	0.142	true	0.289
reiseg1	13	9	true	0.103	true	0.303
ExampleStateView	13	10	false	0.073	false	0.340
frame_manufact	13	10	true	0.191	true	0.141
holloway1	13	10	true	0.942	true	0.238
pnbrexpl_02	13	10	true	0.085	true	0.138
fernandez1	13	11	true	0.161	true	0.395
hee2	13	11	true	0.170	true	0.482
lnet_p5n2	13	11	true	0.230	true	0.220
pnbrexpl_08	13	11	true	0.099	true	0.177
pn_campos_silva2	13	12	true	0.082	true	0.460
pnbrexpl_15	13	12	true	0.278	true	0.230
silva4v2	13	12	true	0.076	true	0.436
voorhoeve1	13	12	true	0.216	true	0.339
board_game	13	13	false	0.107	false	0.345
hack1	13	13	true	0.103	true	0.367
pn_campos_silva7	13	13	true	0.379	true	0.349
pnbrexpl_04	13	13	true	0.134	true	0.369
silva7	13	13	true	0.119	true	0.370
maruster1	13	14	true	0.133	true	0.273
esparza3	13	15	true	0.144	true	0.413
girault3	14	9	true	0.322	true	0.447
philosophers_2	14	10	true	0.110	true	0.491
philosophers_2_rev_2	14	10	true	0.115	true	0.541
pn_silva_03	14	10	true	0.295	true	0.351
rejers1	14	11	true	0.367	true	0.136
hee1	14	12	true	0.489	true	0.609
voorhoeve2	14	12	true	0.439	true	0.357
pnbrexpl_12	14	13	true	0.093	true	0.306
vanDerAalst3	14	13	true	0.130	true	0.146
vanDerAalst7	14	13	true	0.070	true	0.207
jeng1	14	16	true	0.063	true	0.310
lnet_p6n2	14	16	false	0.087	false	0.243
lnet_p9n1	14	16	true	0.206	true	0.301
desel1	15	9	true	0.064	true	0.156
dining_philosophers	15	10	false	0.071	false	0.520
campos3	15	11	true	0.134	true	0.594
oil_separator_cover_s_net_v2	15	11	true	0.394	true	0.328
IEC	15	12	true	0.114	true	0.184
esparza2	15	13	true	2.282	true	0.658
lasire1	15	13	true	0.123	true	0.209
lnet_p8n4	15	16	true	0.084	true	0.527
dongen1	15	17	true	0.070	true	0.422
2pusher	15	18	false	0.114	false	0.542
gaubert1	16	8	true	0.270	true	0.487
silva5	16	8	true	1.449	true	0.667
girault2	16	10	true	0.174	true	0.606
brenner1	16	12	false	0.098	false	0.495
beverage	16	13	true	0.097	true	0.231

beverage_production_part2	16	13	true	0.088	true	0.232
maruster4	16	13	true	0.169	true	0.229
mixer_one_cup	16	13	true	0.080	true	0.218
pnbrexpl_09	16	13	true	0.234	true	0.275
chiola1	16	15	true	0.079	true	0.636
state_space16	16	16	true	0.102	true	0.829
handling_of_incoming_order	17	14	true	0.090	true	0.376
eshuis1	17	15	true	0.133	true	0.395
automation3	17	16	false	0.086	false	0.833
s_net_frame_manufact_quality~	17	16	true	0.130	true	0.225
li1	17	17	true	0.548	true	0.247
pn_campos_silva6	18	11	false	0.146	false	0.699
barkaoui3	18	13	true	0.105	true	0.827
balduzzi1	18	16	false	0.110	false	0.766
vanDerAalst4	18	16	true	0.153	true	0.350
kavi1	18	20	true	0.227	true	0.516
PWM_patterns	19	11	true	0.296	true	0.344
hulgaard1	19	12	true	11.882	true	1.194
mixer	19	15	true	0.109	true	0.324
s_net_frame_manufact_quality~	19	18	true	0.132	true	0.344
maruster2	20	14	true	0.166	true	0.569
holliday1	20	15	true	0.283	true	0.754
philosophers_5	20	15	true	0.128	true	0.625
beverage_production	20	16	true	0.161	true	0.364
miling_machine	20	16	true	0.142	true	0.200
milling	20	16	true	0.170	true	0.313
mixer_mod1	20	16	true	0.141	true	1.058
dingle1	20	20	true	0.103	true	0.965
basile1	21	17	true	0.151	true	0.431
lnet_p8n3	21	17	true	0.174	true	0.298
fernandez3	21	20	true	0.229	true	1.379
pn_fernandez3	21	20	true	0.206	true	0.711
heiner1	22	20	true	0.143	true	1.149
chrzastowski2	22	21	true	0.107	true	0.214
vanDerAalst5	23	23	true	0.178	true	0.656
adam1	24	12	true	1.829	true	3.453
xie1	24	26	true	1.289	true	0.636
HAN	24	40	true	0.105	true	1.719
oil_separator_cover_alfa_net	27	21	true	1.249	true	0.790
lnet_p8n2	28	17	true	0.429	true	1.670
zuberek3	29	21	true	0.575	true	0.708
alfa_net_copy_milling_quality	29	25	false	0.384	false	0.642
ConsistentExample	29	25	false	56 220.855	false	934.609
oil_separator_cover_s_net	29	25	true	0.413	true	0.545
zuberek4	30	21	true	1.944	true	0.945
zuberek1	30	22	true	12.602	true	3.973
well_built_alfa_net_subprocess	30	25	true	0.267	true	9.457
alfa_net_copy_milling_subprocess	30	26	false	0.368	false	0.586
s_net_copy_milling_subprocess	31	28	true	0.521	true	0.635
crossroadSM_FPGA	32	12	true	891 401.325	true	1.577

alfa_net_copy_milling_synchr	32	27	true	0.367	true	1.527
s_net_copy_milling_quality	32	29	false	0.444	false	0.642
state_space32	32	32	true	0.282	true	16.723
vanDerAalst8	34	27	true	0.423	true	8.872
lnet_p4n1	37	41	true	0.172	true	3.571
zuberek5	41	31	n/a	n/a	true	2.726
lnet_p7n1	41	32	true	0.638	true	2.131
state_space48	48	48	true	0.387	true	41.688
PWM_extended	49	31	true	2.510	true	12.776
lnet_p8n1	51	40	false	1.080	false	2.678
cn_crr7	56	15	n/a	n/a	true	3.862
cn_crr10	80	21	n/a	n/a	true	11.128
cn_crr15	120	31	n/a	n/a	true	45.246
cn_crr25	200	51	n/a	n/a	true	294.228